# Vehicle Propulsion Systems
# Lecture 6
## Supervisory Control Algorithms

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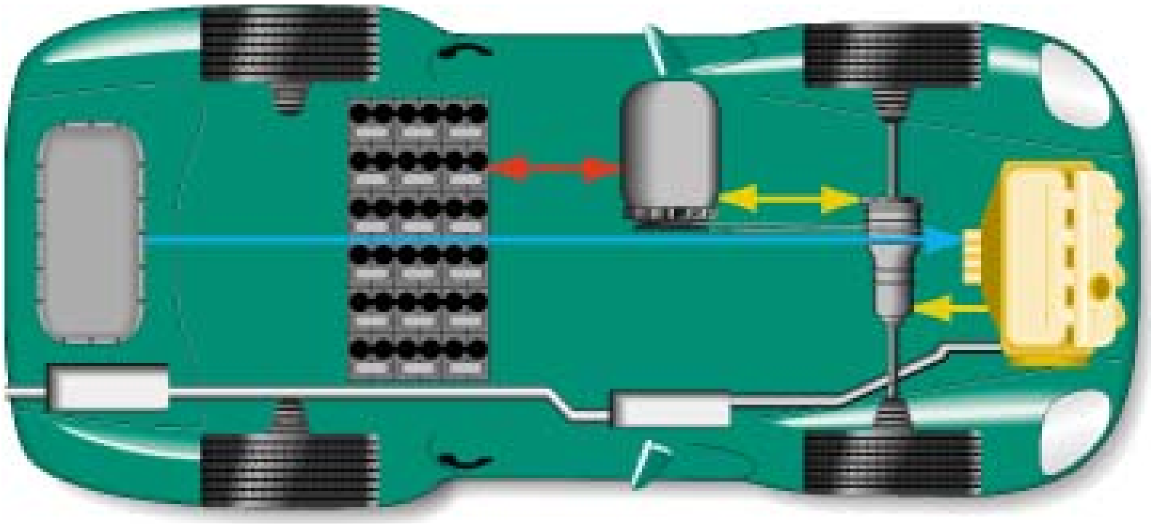
## Outline

## Hybrid Electrical Vehicles – Parallel

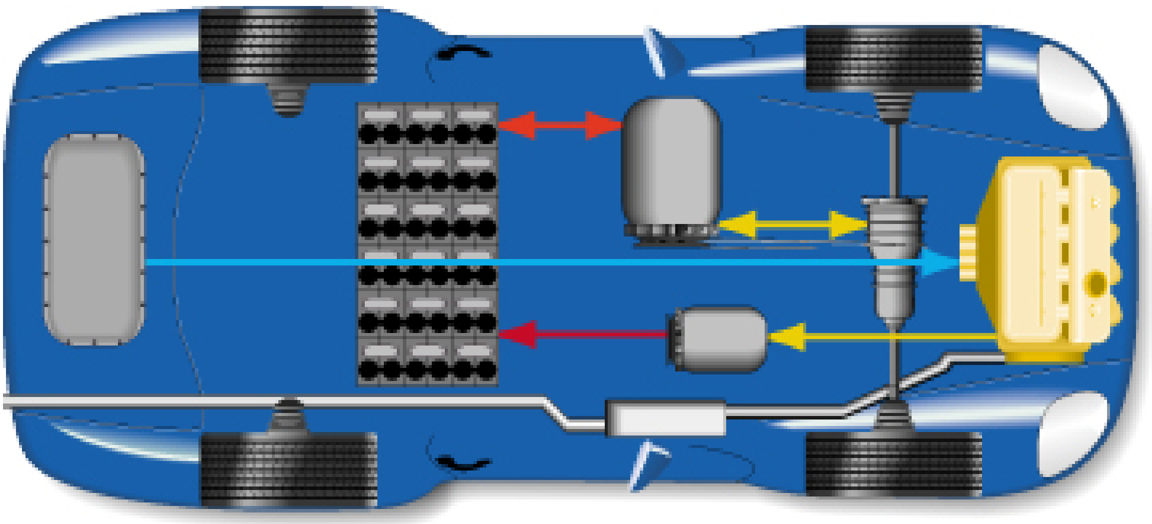
- Two parallel energy paths
- One state in QSS framework, state of charge





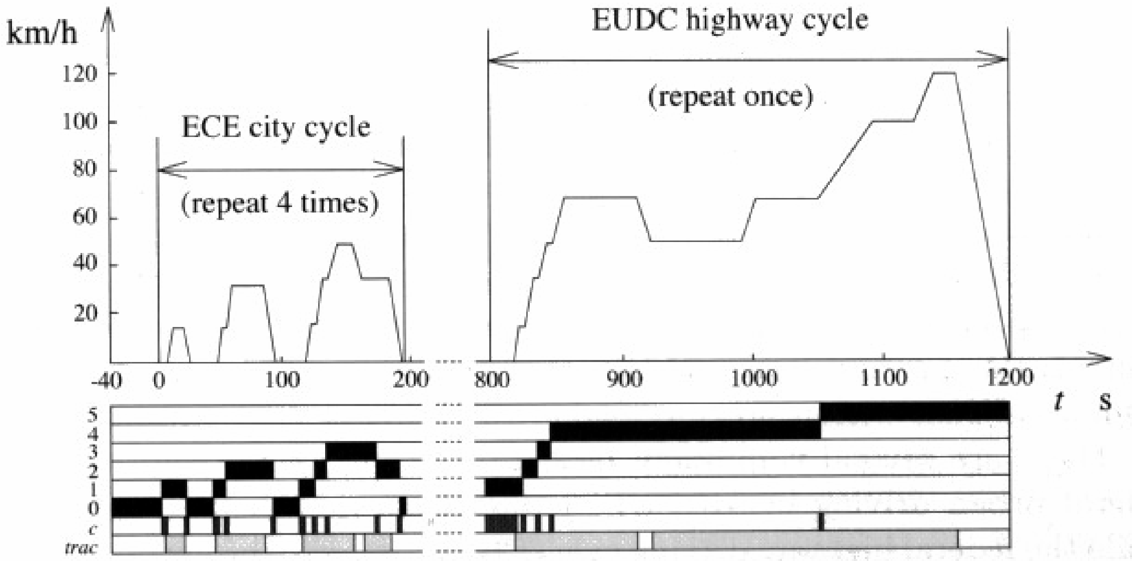
## Hybrid Electrical Vehicles – Serial

- One path; Operation decoupled through the battery
- Two states in QSS framework, state of charge & Engine speed



## Optimization

What gear ratios give the lowest fuel consumption for a given drivingcycle?
–Problem presented in appendix 8.1




Problem characteristics

- Countable number of free variables, $i_{g,j}$, $j \in [1,5]$
- A "computable" cost, $m_f(\cdots)$
- A "computable" set of constraints, model and cycle
- The formulated problem

$$\min_{i_{g,j},\, j \in [1,5]} \quad m_f(i_{g,1}, i_{g,2}, i_{g,3}, i_{g,4}, i_{g,5})$$



## Optimal Control – Problem Motivation

Car with gas pedal $u(t)$ as control input:
How to drive from A to B on a given time with minimum fuel consumption?

- Cost function $\int_0^{t_f} \dot{m}_f(t)dt$
  Fuel mass-flow model $\dot{m}_f = L(v(t), u(t))$ (engine efficiency)
- Infinite dimensional decision variable $u(t)$
- Constraints:
  - Differential equations: Model of the car (the vehicle motion equation)

$$\begin{aligned} m_v \tfrac{d}{dt}v(t) &= F_t(v(t), u(t)) \quad -(F_a(v(t)) + F_r(v(t)) + F_g(x(t))) \\ \tfrac{d}{dt}x(t) &= v(t) \end{aligned}$$

  - Starting point $x(0) = A$
  - End point $x(t_f) = B$
  - Limited control action $0 \le u(t) \le 1$
  - Speed limits $v(t) \le g(x(t))$

## General problem formulation

- Cost function (a functional)

$$J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t)dt$$

- Dynamic system model (constraints)

$$\frac{d}{dt}x = f(x(t), u(t), t), x(t_a) = x_a$$

- Control and state (path) constraints

$$u(t) \in U(t)$$

$$x(t) \in X(t)$$



## Dynamic programming – Problem Formulation

- Optimal control problem

$$\begin{aligned} \min J(u) &= \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t)dt \\ s.t.\ \frac{d}{dt}x &= f(x(t), u(t), t) \\ x(t_a) &= x_a \\ u(t) &\in U(t) \\ x(t) &\in X(t) \end{aligned}$$

- $x(t)$, $u(t)$ functions on $t \in [t_a, t_b]$
- Search an approximation to the solution by discretizing
  - the state space $x(t)$
  - and maybe the control signal $u(t)$

  in both amplitude and time.
- The result is a combinatorial (network) problem

## Deterministic Dynamic Programming – Basic algorithm

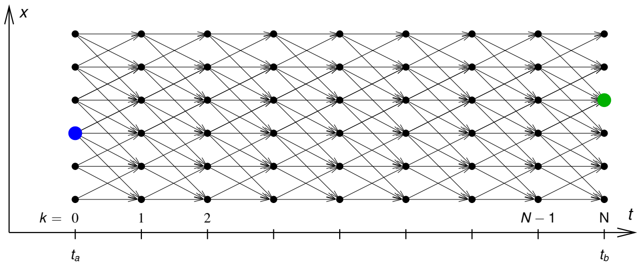
Discretize the time and state space, and search for an approximation to the solution.

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$
$$x_{k+1} = f_k(x_k, u_k)$$

Guarantees a global solution, within the grid.

**Algorithm idea**

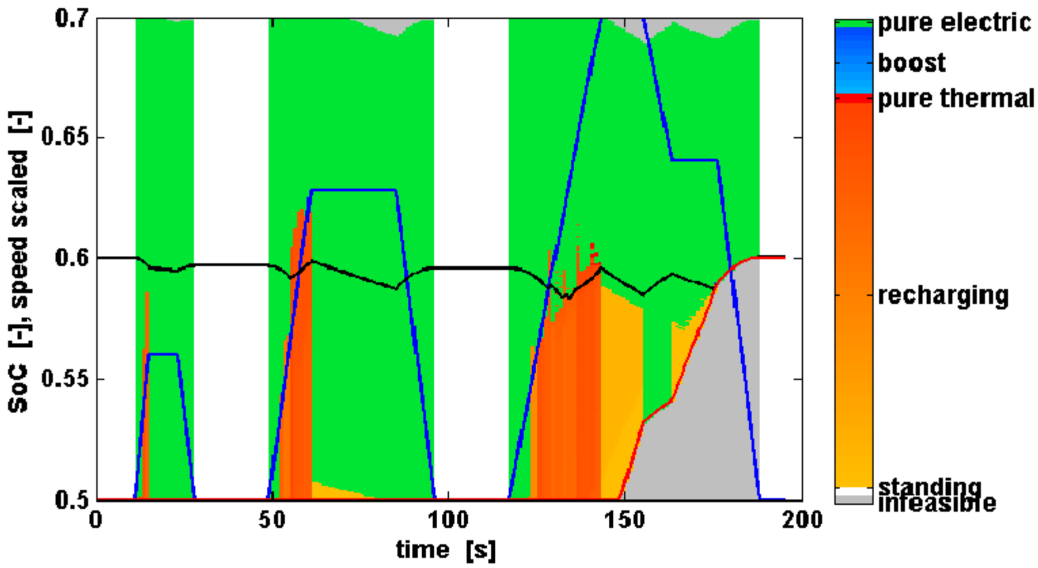
Start at the end and proceed backwards in time to build up an optimal cost-to-go function, store the corresponding control signal.






## Parallel Hybrid Example

- Fuel-optimal torque split factor $u(SOC, t) = \frac{T_{e-motor}}{T_{gearbox}}$
- ECE cycle
- Constraints $SOC(t = t_f) \geq 0.6$, $SOC \in [0.5, 0.7]$



## Outline



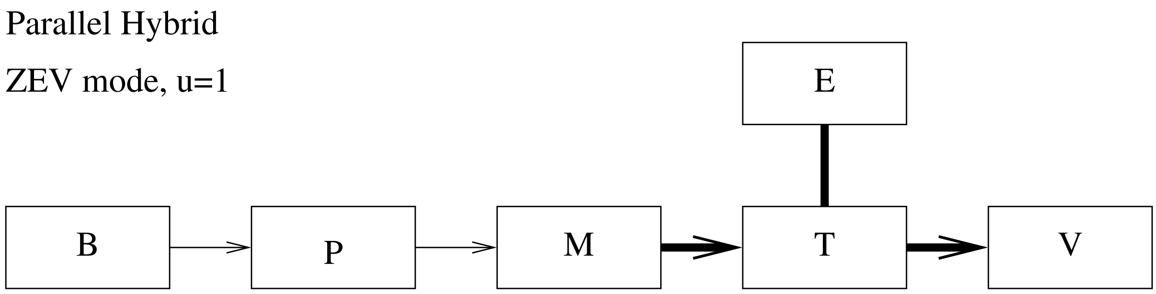


## Parallel Hybrid – Modes and Power Flows

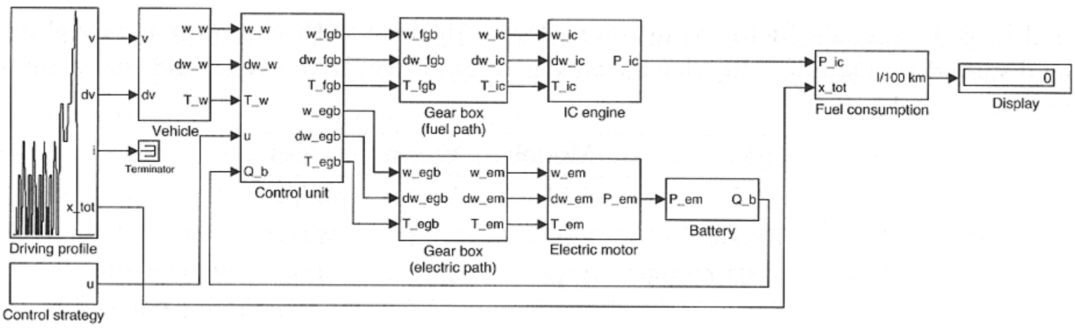
The different modes for a parallel hybrid

$$u \approx P_{batt}/P_{vehicle}$$

Battery drive mode (ZEV)



# Control algorithms




- Determining the power split ratio $u$

$$u_j(t) = \frac{P_j(t)}{P_{m+1}(t) + P_l(t)} \quad (4.110)$$

- Clutch engagement disengagement $B_c \in \{0, 1\}$
- Engine engagement disengagement $B_e \in \{0, 1\}$



# Strategies for the Parallel Hybrid

Power split $u$, Clutch $B_c$, Engine $B_e$

| | Mode | $u$ | $B_e$ | $B_c$ |
|---|---|---|---|---|
| 1 | ICE | 0 | 1 | 1 |
| 2a | ZEV | 1 | 0 | 0 |
| 2b | ZEV | 1 | 0 | 1 |
| 3 | Power assist | [0,1] | 1 | 1 |
| 4 | Recharge | $< 0$ | 1 | 1 |
| 5a | Regenerative braking | 1 | 0 | 0 |
| 5a | Regenerative braking | 1 | 0 | 1 |

All practical control strategies have engine shut off when the torque at the wheels are negative or zero; standstill, coasting and braking.

## Classification I – Supervisory Control Algorithms

- Non-causal controllers
  - Detailed knowledge about future driving conditions.
  - Position, speed, altitude, traffic situation.
  - Uses:
    Analyses of optimal behavior on regulatory drive cycles
    Public transportation, long haul operation, GPS based route planning.
- Causal controllers
  - No knowledge about the future.
  - Use information about the current state.
  - Uses:
    “The normal controller”, on-line, in vehicles without planning



## Classification II – Vehicle Controllers

- Heuristic controllers
  –Causal
  –State of the art in most prototypes and mass-production
- Optimal controllers
  –Often non-causal
  –Some causal solutions exist, ECMS.
- Sub-optimal controllers
  –Uses optimization to solve a smaller sub-problems
  –Often causal.

On-going work to include optimal controllers in production vehicles.

## Some Comments About the Problem

- Important problem for the industry – Area of competition
- Difficult problem
- Unsolved problem for causal controllers
- Rich body of
  engineering reports and
  research papers on the subject

  –This can clearly be seen when reading chapter 7!
  It has been the main research area for Lino Guzzella and Antonio Sciarretta.



## Outline

## Heuristic Control Approaches

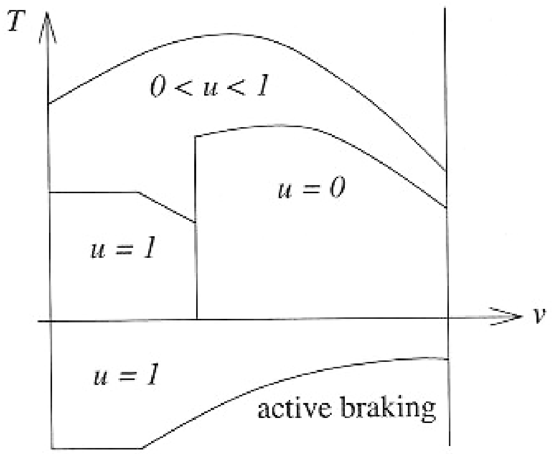
Operation usually depends on a few vehicle operation

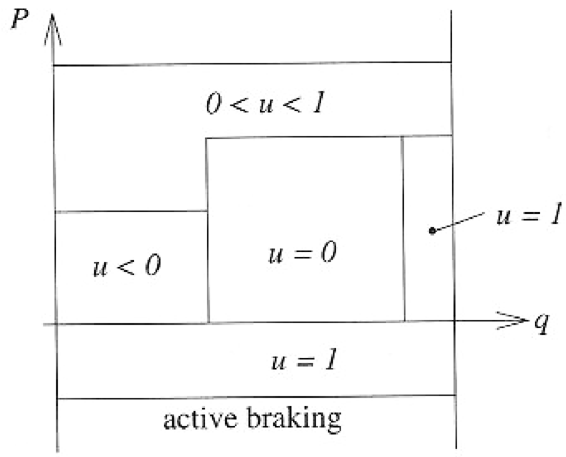
- Rule based:
  Nested `if-then-else` clauses
  `if` $v < v_{low}$ `then` use electric motor (u=1).
  `else...`
- Fuzzy logic based
  Classification of the operating condition into fuzzy sets.
  Rules for control output in each mode.
  Defuzzyfication gives the control output.



## Heuristic Control Approaches

- Parallel hybrid vehicle (electric assist)




- Determine control output as function of some selected state variables: vehicle speed, engine speed, state of charge, power demand, motor speed, temperature, vehicle acceleration, torque demand.

## Heuristic Control Approaches – Concluding Remarks

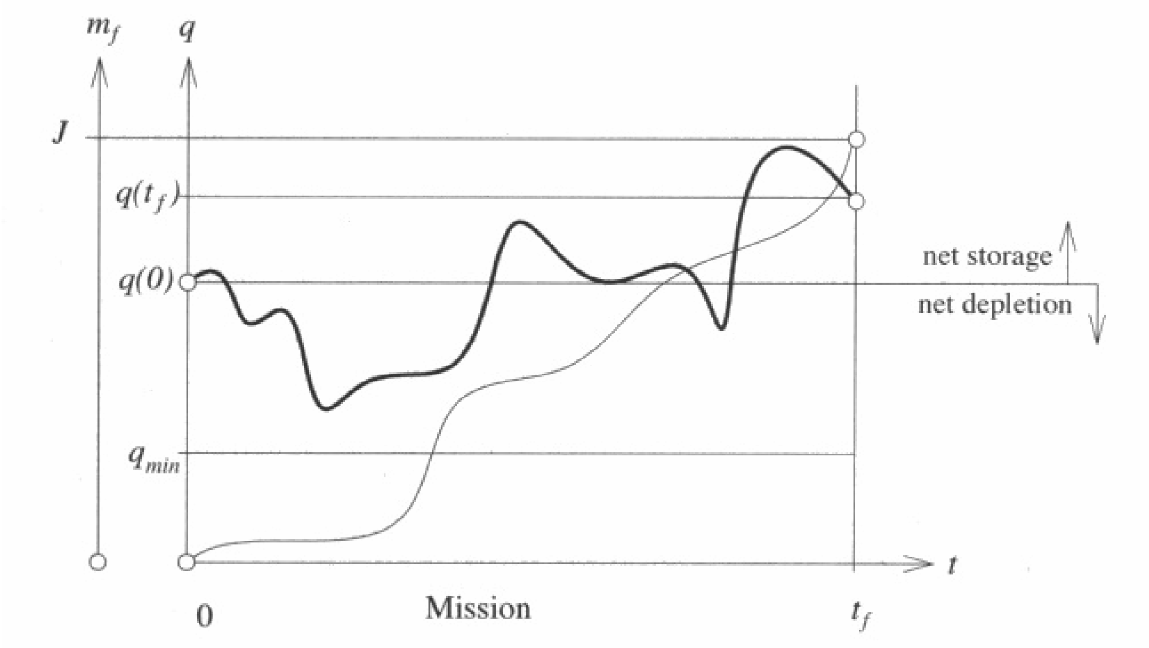
- Easy to conceive
- Relatively easy to implement
- Result depends on the thresholds
- Proper tuning can give good fuel consumption reduction and charge sustainability
- Performance varies with cycle and driving condition
  –Not robust
- Time consuming to develop and tune for advanced hybrid configurations



## Outline

# Consider a driving mission

- Variables.
  Control signal – $u(t)$, System state – $x(t)$, State of charge - $q(t)$ (is a state).






# Formulating the Optimal Control Problem

–What is the optimal behaviour? Defines *Performance index J*.

- Minimize the fuel consumption

$$J = \int_0^{t_f} \dot{m}_f(t, u(t))dt$$

- Balance between fuel consumption and emissions

$$J = \int_0^{t_f} \Big[\dot{m}_f(t, u(t)) + \alpha_{CO}\dot{m}_{CO}(x(t), u(t)) + \alpha_{NO}\dot{m}_{NO}(x(t), u(t)) + \alpha_{HC}\dot{m}_{HC}(x(t), u(t))\Big] dt$$

- Include driveability criterion

$$J = \int_0^{t_f} \dot{m}_f(t, u(t)) + \beta \left(\frac{d}{dt}a(t)\right)^2 dt$$

## Towards a Solution to the Problem

In the course we are focusing on the fuel consumption.

- Minimize the fuel consumption

$$J = \int_0^{t_f} \dot{m}_f(t, u(t))dt$$

- The driving cycle is specified, no freedom
- Our freedom is in the choice of how to use the electric energy in the battery
- The focus is also on hybrid vehicles that need to be charge sustaining
  – Constraint $q(0) = q(t_f)$
- Plugin Hybrid Electric Vehicles (PHEV) can be treated similarly, where the discharge profile is specified.



## Including the constraint

- Hard or soft constraints

$$\min J(u) = \int_0^{t_f} L(t, u(t))dt$$
$$s.t.\ q(0) = q(t_f)$$

$$\min\ J(u) = \phi(q(t_f)) + \int_0^{t_f} L(t, u(t))dt$$

- How to select $\phi(q(t_f))$?

$$\phi(q(t_f)) = \alpha\,(q(t_f) - q(0))^2$$

  penalizes high deviations more than small, independent of sign

$$\phi(q(t_f)) = w\,(q(0) - q(t_f))$$

  penalizes battery usage, favoring energy storage for future use
- One more feature from the last one

## Including the constraint

- Including battery penalty according to

$$\phi(q(t_f)) = w\,(q(0) - q(t_f)) = -w \int_0^{t_f} \dot{q}(t)dt$$

  enables us to rewrite

$$\min \; J(u) = \int_0^{t_f} L(t, u(t)) - w\,\dot{q}(t)dt$$

- Note the similarity to the method of using Lagrange multiplier.



## Constraints That are Also Included

- State equation $\dot{x} = f(x)$ is also included
- We are considering a parallel hybrid with only one state, the SoC (or equivalently $q(t)$)

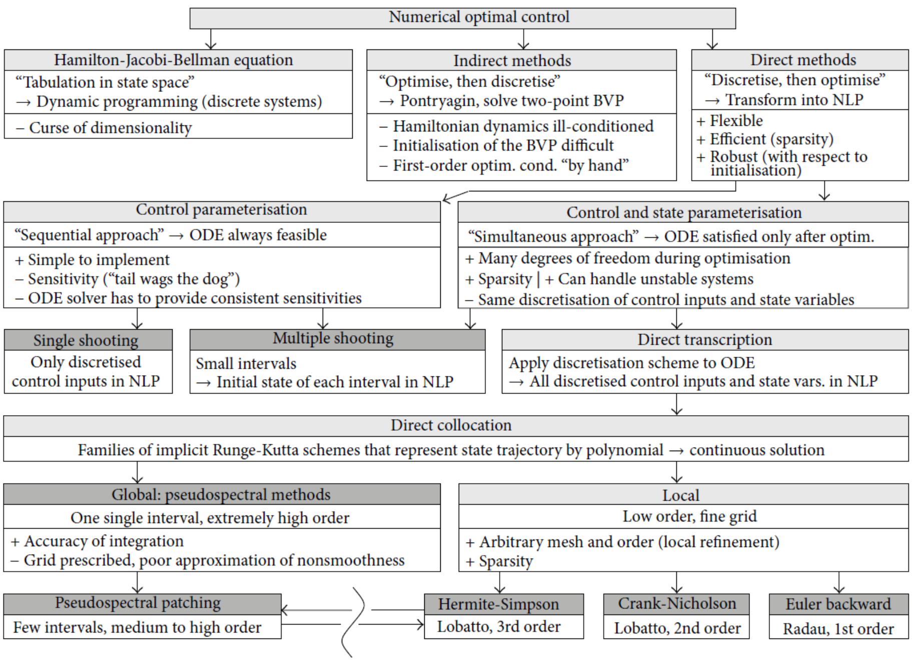
$$\begin{aligned} \min \; & J(u) = \phi(q(t_f), t_f) + \int_0^{t_f} L(t, u(t))dt \\ s.t. \; & \frac{d}{dt}q = f(t, q(t), u(t)) \\ & u(t) \in U(t) \\ & q(t) \in Q(t) \end{aligned}$$

# Outline





# Numerical Methods for Solving Optimal Control Problems




Overview from Jonas Asprion, "Optimal Control of Diesel Engines, Modeling, Numerical Methods, and Applications", PhD Thesis, ETH, (2015).

**Commercial Break**

Course TSRT08 Optimal Control

## Analytical Solutions to Optimal Control Problems

- A general optimal control problem formulation

$$\min J(u) = \phi(x(t_f), t_f) + \int_0^{t_f} L(t, u(t))dt$$
$$s.t.\ \dot{x}(t) = f(t, x(t), u(t))$$

- Hamiltonian defined in optimal control theory

$$H(t, x(t), u(t), \lambda(t)) = L(t, u(t)) + \lambda(t) f(t, x(t), u(t))$$

- $\lambda(t)$ is a Lagrange multiplier, it's a dear child with many names
  - Lagrange variable
  - Adjoint state
  - Co-state
  - Most often denoted $\lambda(t)$, but $\mu(t)$ is also used.



## Analytical Solutions to Optimal Control Problems

- Hamiltonian

$$H(t, x(t), u(t), \lambda(t)) = L(t, x(t)) + \lambda(t) f(t, x(t), u(t))$$

- Necessary conditions for optimality

$$\dot{x}(t) = f(t, x(t), u(t))$$
$$\dot{\lambda}(t) = -\frac{\partial}{\partial x} H(t, x(t), u(t), \lambda(t))$$

- At the optimum $x^*(t), u^*(t), \lambda^*(t)$

$$H(x^*(t), u^*(t), \lambda^*(t)) \leq H(x^*(t), u(t), \lambda^*(t))$$

- Pontryagin's Minimum/Maximum Principle

$$u^*(t) = \arg\min_{u(t)} H(x^*(t), u(t), \lambda^*(t))$$

Remaining question: What can we do to find $\lambda^*(t)$?

## Modeling a Parallel HEV in a Driving Cycle

The cycle is given so the propulsive power demand $P_p(t)$ from the Powertrain is given.

- We want to minimize the fuel energy, i.e. integral of the power $P_f(t)$.
- We have the freedom to use electrochemical energy from the battery $P_{ech}(t) = U(t)\, I(t)$, this is our control signal $u(t)$.
- The problem formulation, with charge sustain strategy becomes

$$
\begin{aligned}
\min\ J(u) &= \int_0^{t_f} P_f(t, u(t))dt \\
s.t.\ \frac{dSoC(t)}{dt} &= -\frac{P_{ech}(t)}{U(SoC(t))\, Q_{tot}} \\
SoC(0) &= SoC(t_f) \\
P_p(T) &= \eta_{eng}\, P_f(t) + \eta_{el}\, P_{ech}(t)
\end{aligned}
$$

where the last algebraic constraint, is the propulsive power demand.



## Energy Management for the Parallel HEV in a Driving Cycle

- Set up the Hamiltonian

$$H(t, SoC(t), u(t), \lambda(t)) = P_f(t, u(t)) - \lambda(t)\frac{P_{ech}(t)}{U(SoC(t))\, Q_{tot}}$$

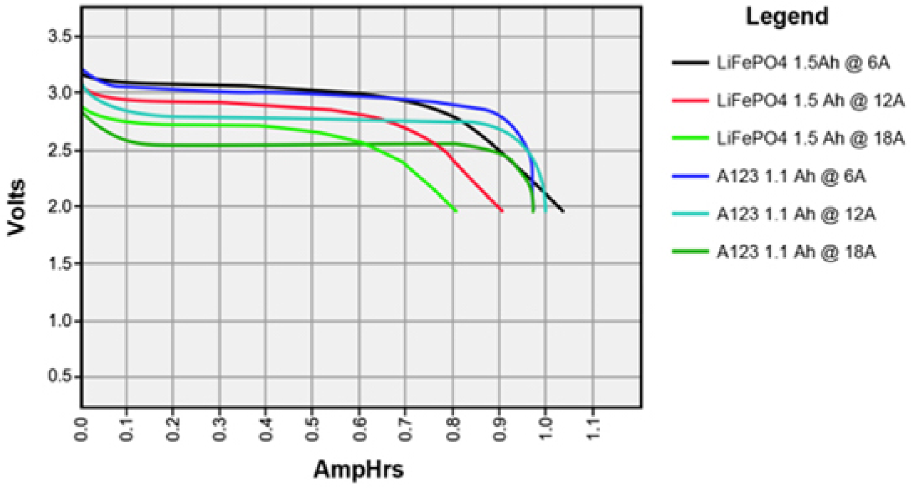
- Now we use the necessary conditions for the adjoint state.

$$\dot{\lambda}(t) = -\frac{\partial}{\partial x}H(t, x(t), u(t), \lambda(t)) = \frac{\partial}{\partial SoC}\frac{P_{ech}(t)}{U(SoC(t))\, Q_{tot}} = -\frac{P_{ech}(t)}{U(SoC(t))^2\, Q_{tot}}\frac{\partial U(SoC(t))}{\partial SoC}$$

- Lets have a look at $\frac{\partial U(SoC(t))}{\partial SoC}$.

## Battery Voltage and SoC

Typical characteristics. Q=SOC has little or no influence in the normal region.




(Source: batteryuniversity.com)

A good model for normal operation is

$$\frac{\partial U(SoC(t))}{\partial SoC} = 0$$

Which gives

$$\dot{\lambda}(t) = 0$$

$\lambda(t)$ thus becomes a constant $\lambda_0$.



## Solution Algorithm

Set up the models for vehicle and engine with fuel flow and the power electronics and electric machine.

1. Setup all equations and form the Hamiltonian.
2. Make a guess on $\lambda_0$.
3. Run a drivcycle simulation with your vehicle where you in each step minimize the Hamiltonian to get the control signal.
4. If the charge sustainability is fulfilled then stop.
5. Modify $\lambda_0$ and go to step 3.

A driving cycle is mapped to a $\lambda_0$.

If we want to use it in normal driving, we don't know $\lambda_0$ and cannot iterate to find it.

## Analytical Solutions to Optimal Control Problems

If we have an incorrect $\lambda_0$ the SoC will drift away from its nominal $SoC_{ref}$ value.

**Solution**

Start with an initial guess then look at SoC and update $\lambda_0$ as we drive, use for example a PI-controller.

$$\lambda_0 = PI(SoC - SoC_{ref})$$

This is called Adaptive ECMS, as it adapts $\lambda_0$ to the driving cycle.



## Analytical Solutions to Optimal Control Problems

- $\mu_0$ depends on the (soft) constraint

$$\mu_0 = \frac{\partial}{q(t_f)}\phi(q(t_f)) = /\text{special case}/ = -w$$

- Different efficiencies

$$\mu_0 = \frac{\partial}{\partial q(t_f)}\phi(q(t_f)) = \begin{cases} -w_{dis}, & q(t_f) > q(0) \\ -w_{chg}, & q(t_f) < q(0) \end{cases}$$

- Introduce equivalence factor (scaling) by studying battery and fuel power

$$s(t) = -\mu(t)\frac{H_{LHV}}{V_b\, Q_{max}}$$

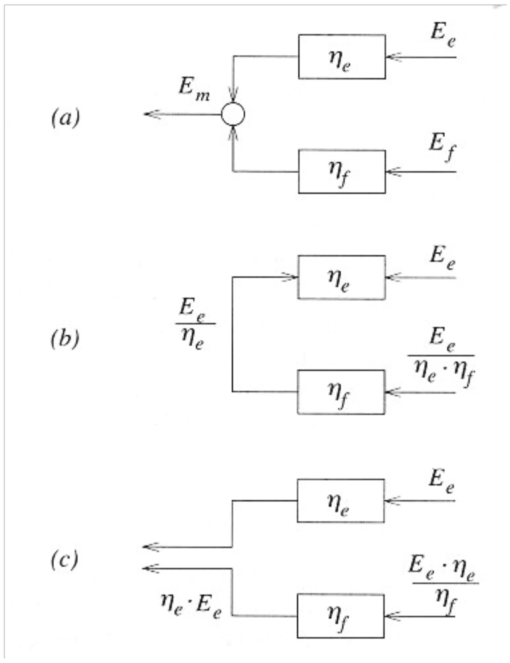
ECMS – Equivalent Consumption Minimization Strategy

# Determining Equivalence Factors I

Constant engine and battery efficiencies

$$s_{dis} = \frac{1}{\eta_e \eta_f}$$

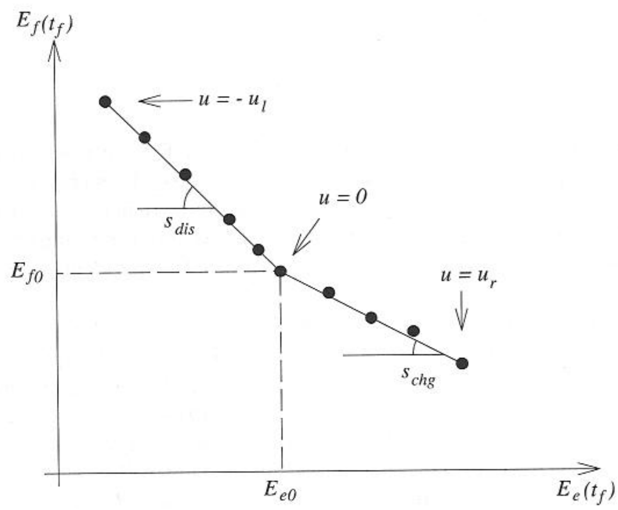
$$s_{chg} = \frac{\eta_e}{\eta_f}$$






# Determining Equivalence Factors II

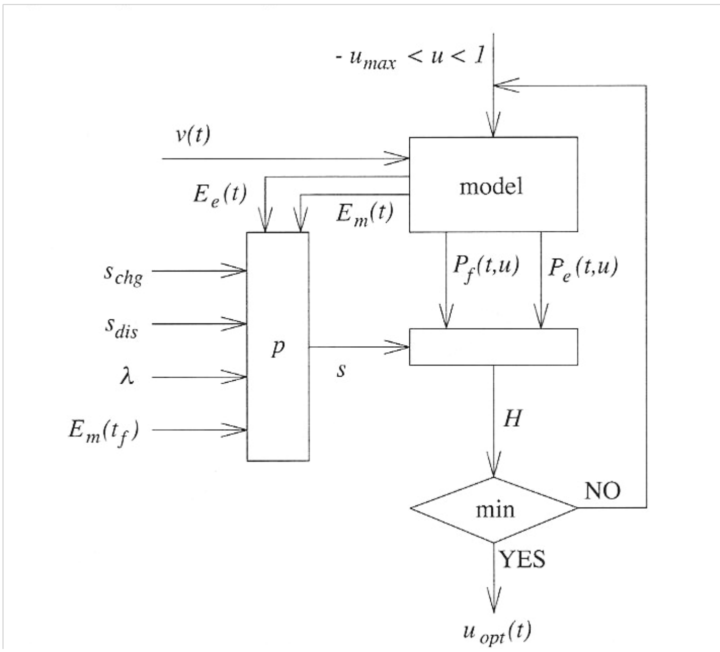
- Collecting battery and fuel energy data from test runs with constant $u$ gives a graph
- Slopes determine $s_{dis}$ and $s_{chg}$.



## ECMS On-line Implementation

Flowchart

There is also a T-ECMS (telemetry-ECMS)

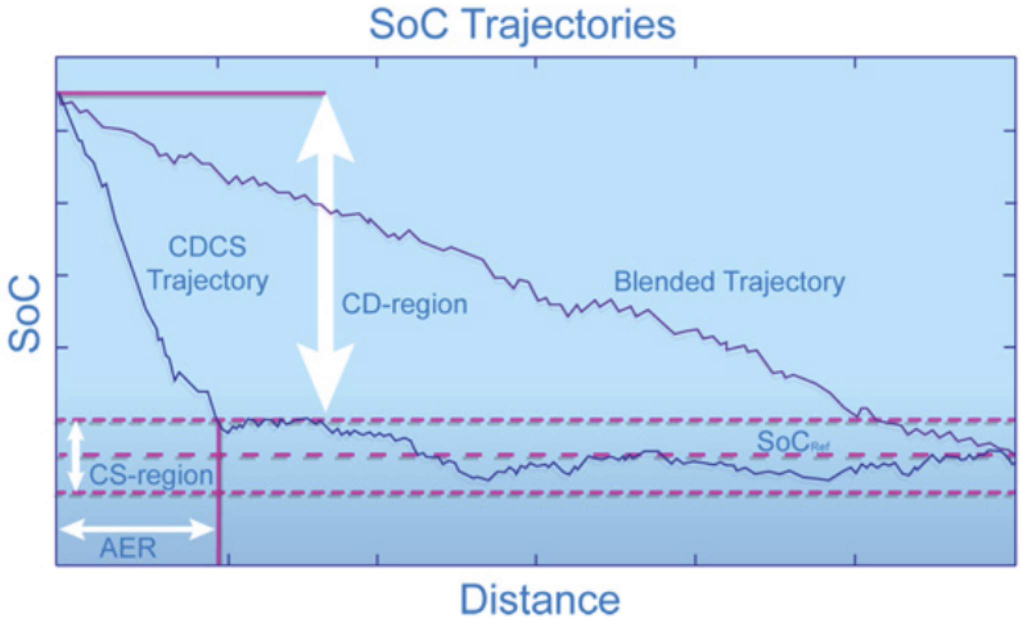





## Outline

## PHEV – Charge Deplete then Charge Sustain (CDCS)

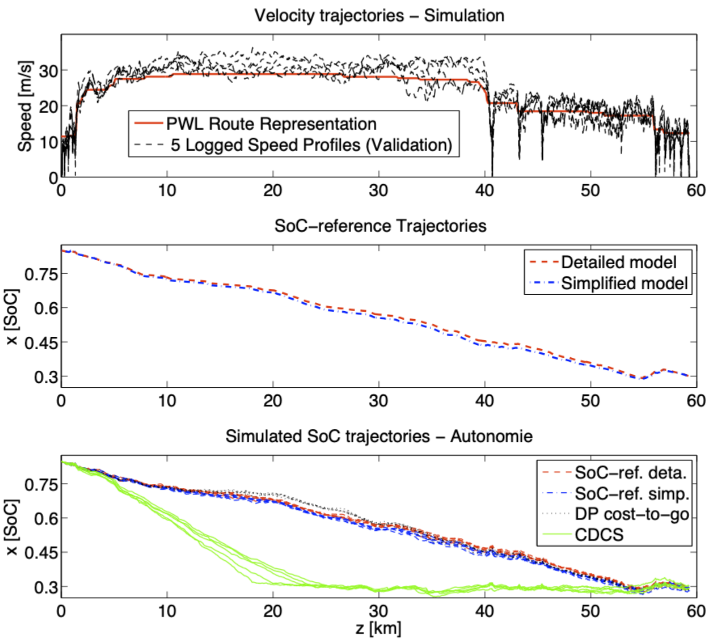
Statistic analysis shows that most trips are short, good idea to use up all electricity.






## PHEV – Blended Mode

- 5 Commuter tracks for a car.
- Compute an $SoC(t)$ reference.
- Use PI controller to follow that $SoC(t)$.
- Use more theory, DP cost to go can give $\lambda$.
- Compare to CDCS.



# PHEV – Comparison

| Simulation Results - All 5 commuter Trips | | | | |
|---|---|---|---|---|
| | $\int \lvert I(t) \rvert dt$ [Ah] | $\int \dot{m}(t)dt$ [kg] | $SoC(t_f)$ (mean) | C-rate (mean) |
| SoC-ref Detailed | 209 | 5.45 | 0.295 | 1.49 |
| Soc-ref Simplified | 210 | 5.43 | 0.291 | 1.49 |
| DP cost-to-go | 211 | 5.41 | 0.287 | 1.50 |
| CDCS | 241 | 5.85 | 0.296 | 1.71 |

6.8%–9.0% Improvements in fuel economy, with blended strategies.

Viktor Larsson, Lars J. Mårdh, Bo Egardt “Comparing Two Approaches to Precompute Discharge Strategies for Plug-in Hybrid Electric Vehicles”, IFAC AAC, Tokyo, 2013.