

TSTE26 Powergrid and technology for renewable
production

Lecture 3
Electric power system

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Outline

Electric Power Systems, A first course by Ned Mohan

- Chapter 2 Review of basic electric circuits (available as preview on the internet)
- Chapter 4 AC transmission lines (pdf in Lisam)

Symbols and Conventions

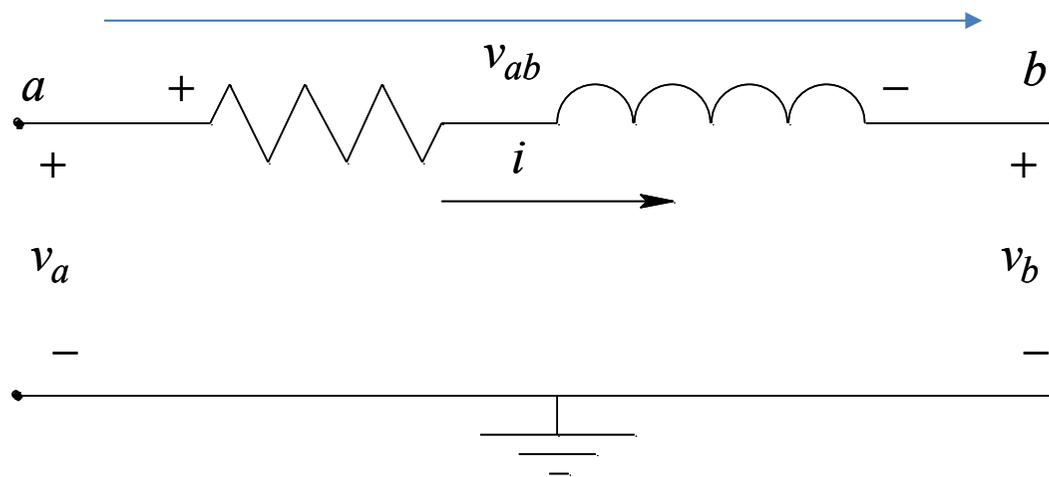


Fig. 2-1 Convention for voltages and currents.

a – Sender

b – Receiver

Phasors

$$v(t) = \sqrt{2} V \cos \omega t$$

$$i(t) = \sqrt{2} I \cos(\omega t - \phi)$$

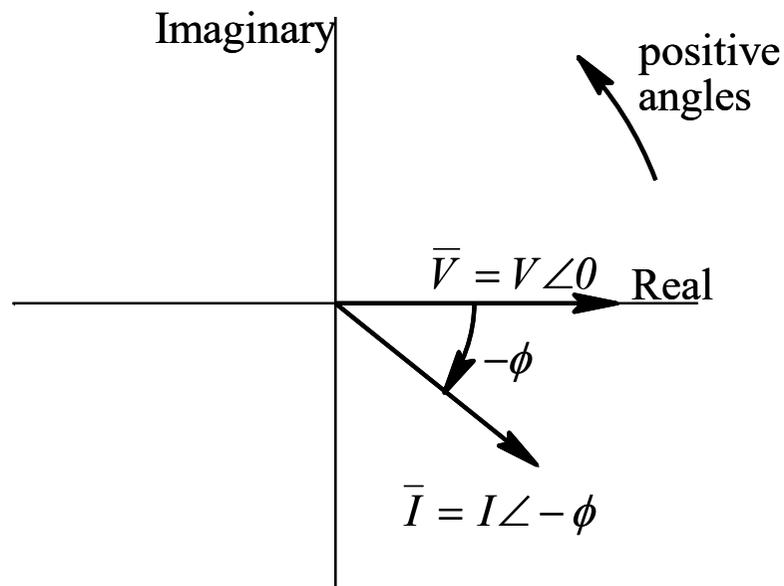


Fig. 2-2 Phasor diagram.

Inductance:
Voltage comes before current

Capacitance:
Current comes before voltage

Phasor Analysis (RMS)

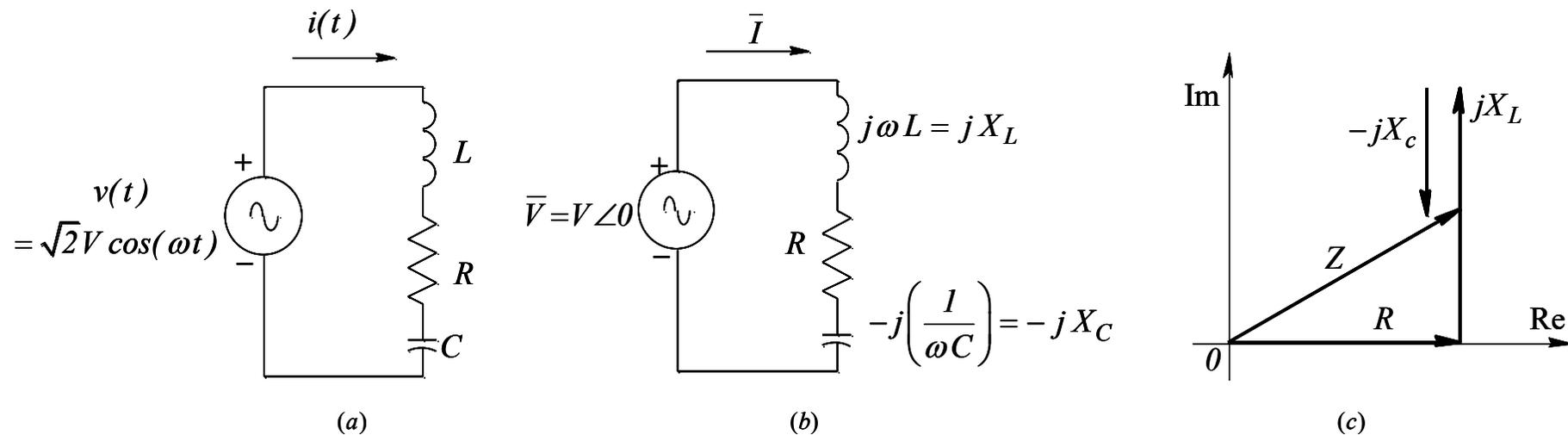


Fig. 2-3 A circuit (a) in time-domain and (b) in phasor-domain; (c) impedance triangle.

$$u_L = L \frac{di}{dt}$$

$$\bar{U}_L = j\omega L \bar{I}$$

$$u_C(t) = u_C(t_1) + \frac{1}{C} \int_{t_1}^t i(t) dt$$

$$\bar{U}_C = \frac{1}{j\omega C} \bar{I}$$

Instantaneous Power Flow

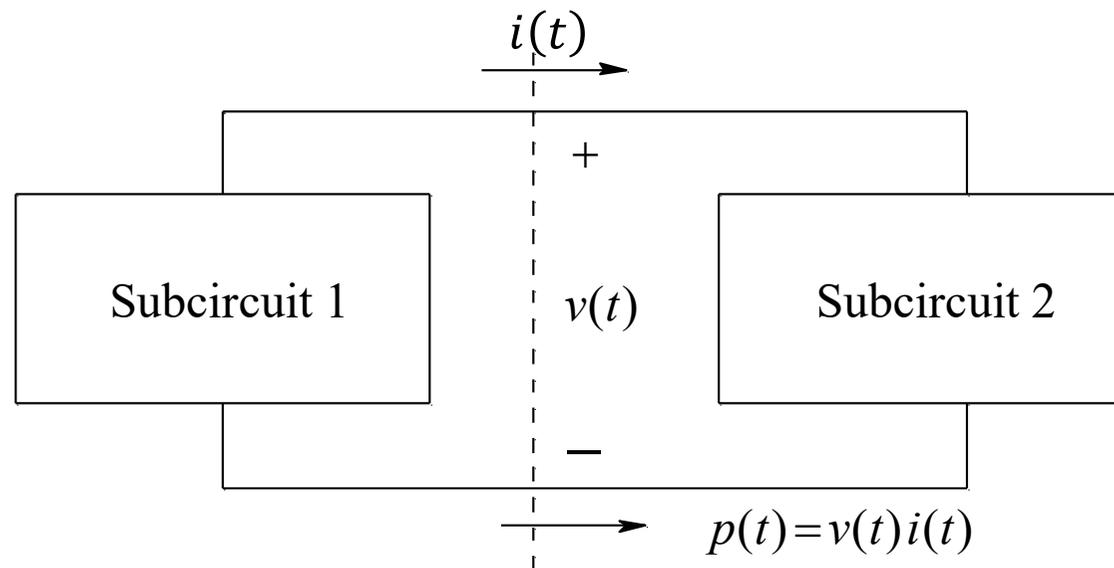
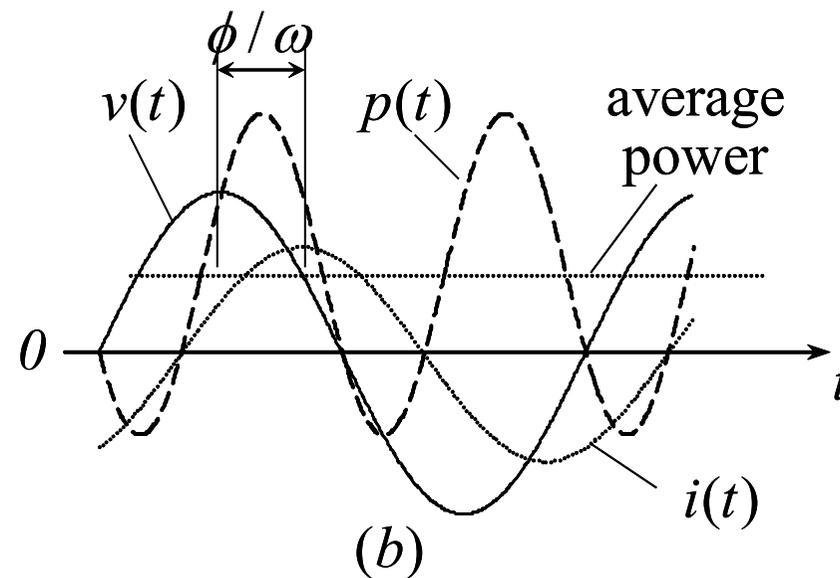
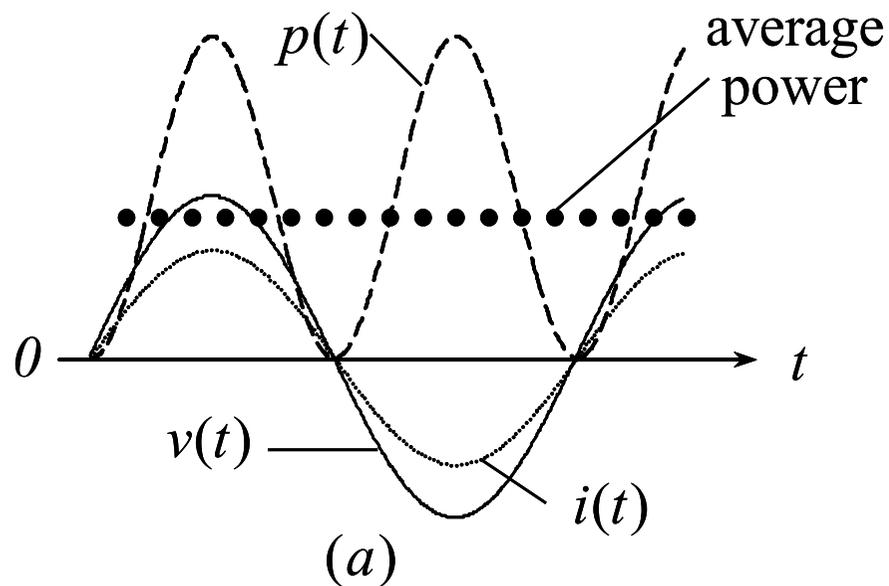


Figure 2-6 A generic circuit divided into two sub-circuits.

Real(Active) Power



$$(a) \quad p(t) = \sqrt{2}V \cos \omega t \cdot \sqrt{2}I \cos \omega t = 2VI \cos^2 \omega t = VI + VI \cos 2\omega t$$

$$(b) \quad p(t) = \sqrt{2}V \cos \omega t \cdot \sqrt{2}I \cos(\omega t - \phi) = VI \cos \phi + VI \cos(2\omega t - \phi)$$

P, Q, VA and Power Factor

$$\bar{S} = \bar{V}\bar{I}^* = \bar{V} \cdot \text{conj}(\bar{I})$$

$$\bar{S} = P + jQ$$

$$P = VI \cos \phi$$

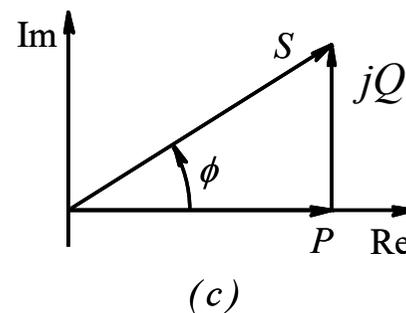
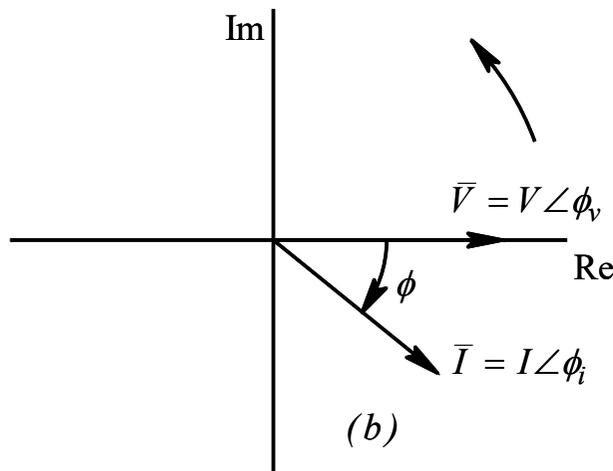
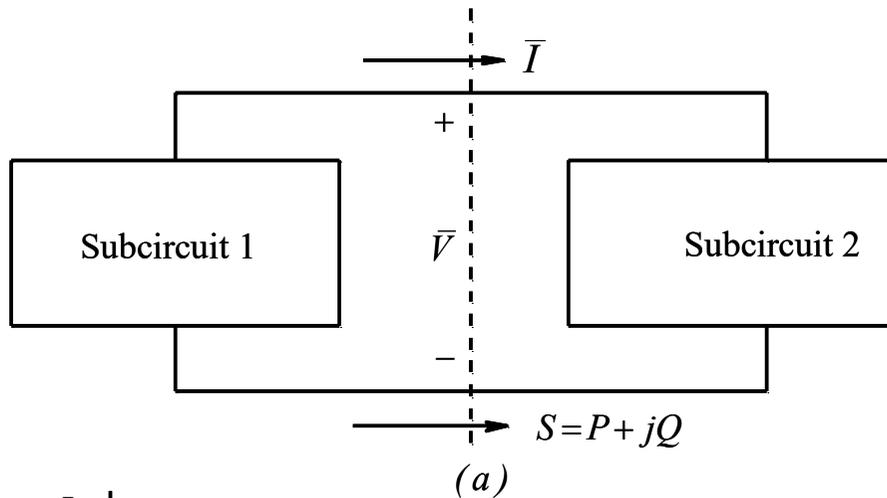
$$Q = VI \sin \phi$$

$$|S| = \sqrt{P^2 + Q^2}$$

$$\phi = \phi_v - \phi_i$$

$$\phi = \tan^{-1} \left(\frac{Q}{P} \right)$$

$$\text{Power Factor} = \frac{P}{VI} = \cos \phi$$



Circuit calculation example

$$V_1 = 120 \angle 0^\circ \text{ V}$$

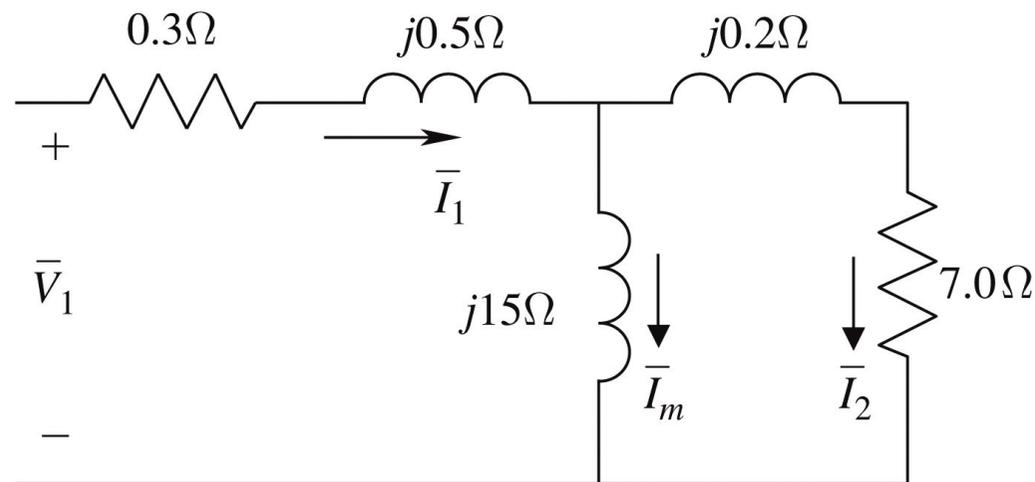


Figure 2.5
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- Calculate the input impedance \bar{Z}_{in} and \bar{I}_1 (Example 2.2)
- Calculate P, Q, S and the power factor (Example 2.3)

Circuit calculation example - solution

```

3  Z1 = 0.3 + 0.5j;
4  Zm = 15j;
5  Z2 = 7 + 0.2j;
6  Zin = Z1 + Zm * Z2 / (Zm + Z2) %5.9 + j3.3 ohm
7
8  V1 = 120;
9  I1 = V1 / Zin %15.5 - j8.6A
10
11 I2 = I1 * Zm / (Zm + Z2)
12 Im = I1 - I2
13 Sm = Zm * Im * conj(Im)
14 S2 = Z2 * I2 * conj(I2)
15 S1 = Z1 * I1 * conj(I1)
16
17 S = S1 + S2 + Sm %1858 + j1031 VA
18 P = real(S)
19 Q = imag(S)
20
21 Sxx = Zin * I1 * conj(I1) %alternative solution
22
23 pf = cosd(atan(Q/P))

```

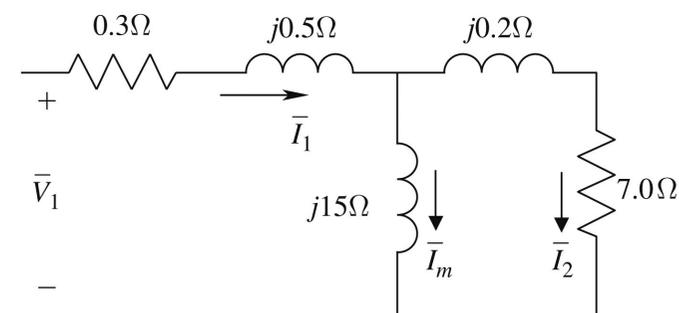


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Example of Power Factor Correction

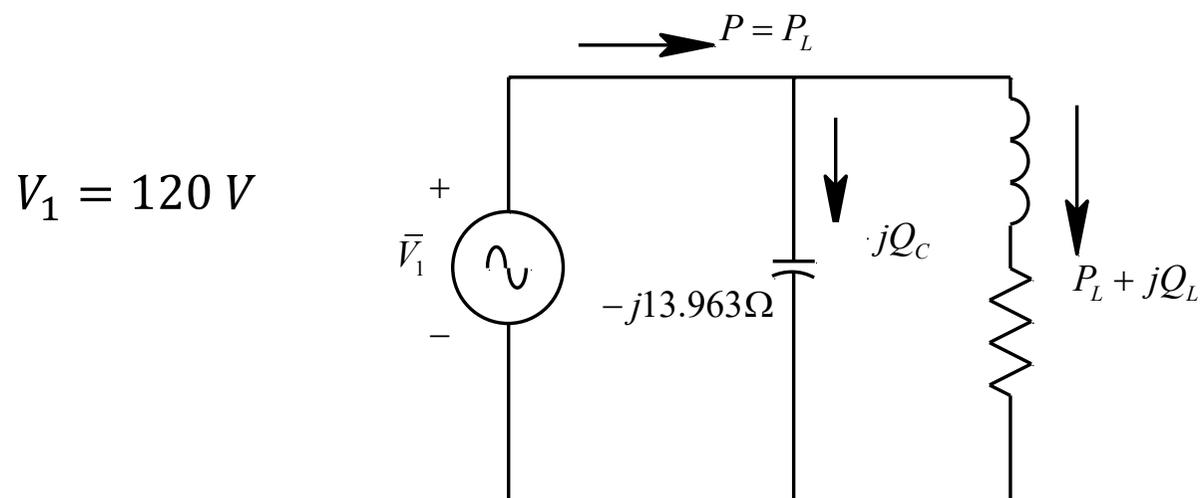


Fig. 2-9 Power factor correction in Example 2-5.

$$P_L + jQ_L = (1858 + j1031)\text{VA}$$

$$\text{Unity power factor, } \cos \phi = 1 \quad Q_L = -Q_c = -\frac{V_1^2}{X_c}$$

Three-Phase Voltages

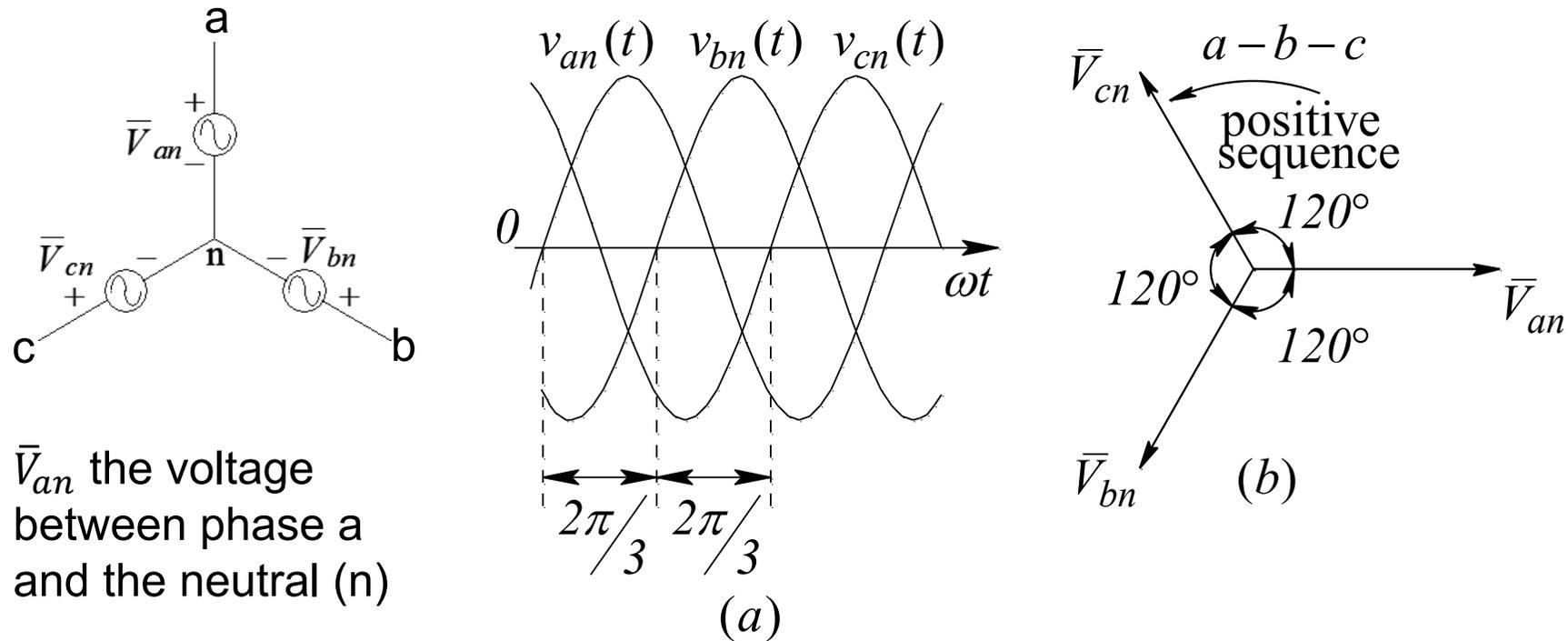


Fig. 2-11 Three-phase voltages in time and phasor domain.

$$\begin{aligned} v_{an} &= \sqrt{2}V \cos(\omega t) \\ v_{bn} &= \sqrt{2}V \cos(\omega t - 120^\circ) \\ v_{cn} &= \sqrt{2}V \cos(\omega t - 240^\circ) \end{aligned}$$

$$\begin{aligned} \bar{V}_{an} &= V \angle 0^\circ \\ \bar{V}_{bn} &= V \angle -120^\circ \\ \bar{V}_{cn} &= V \angle -240^\circ \end{aligned}$$

V_{an} is the RMS phase to neutral voltage

Balanced Three-Phase Circuit Analysis

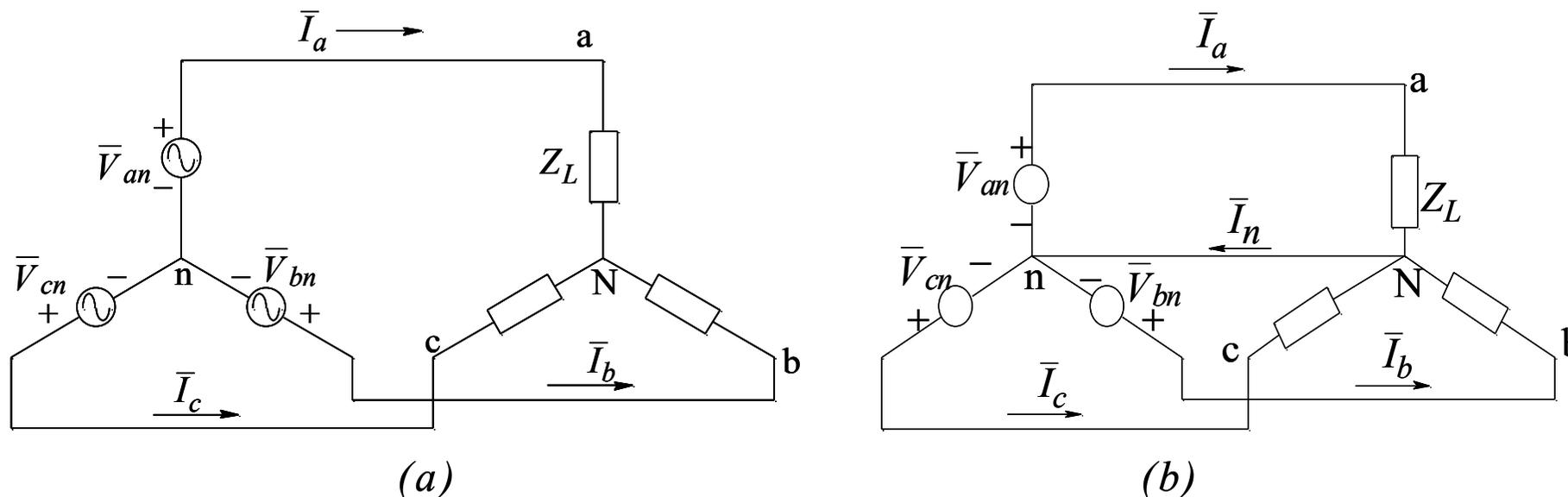


Fig. 2-12 Balanced wye-connected, three-phase circuit.

If the source is symmetrical 3-phase and the load same in all phases.

- $V_n = V_N$
- $I_n = 0 \rightarrow I_a + I_b + I_c = 0$

Per-Phase Analysis

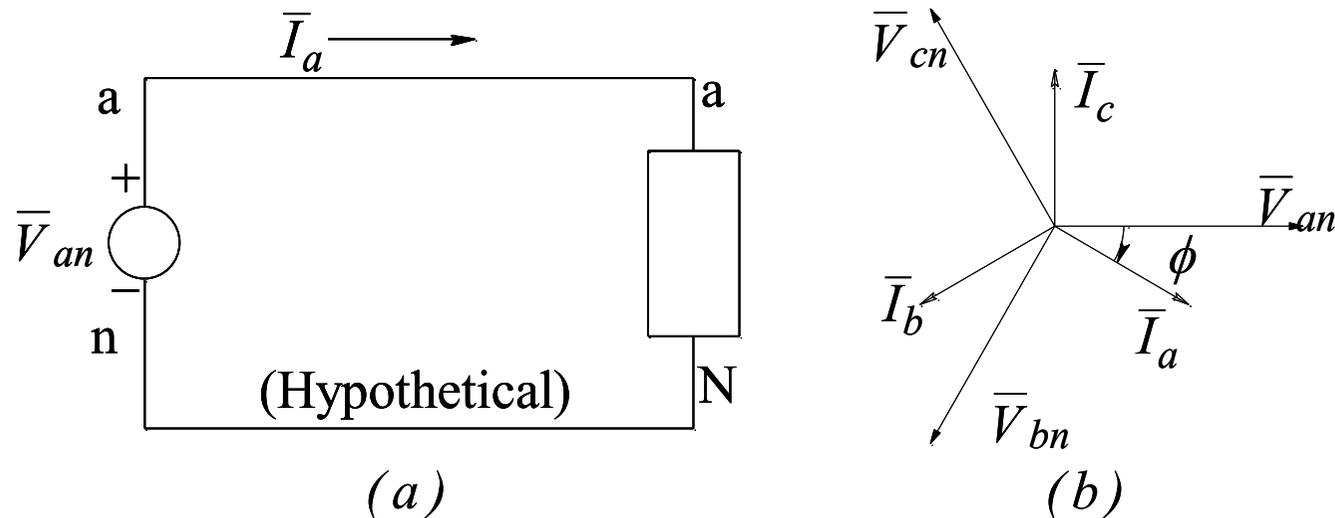
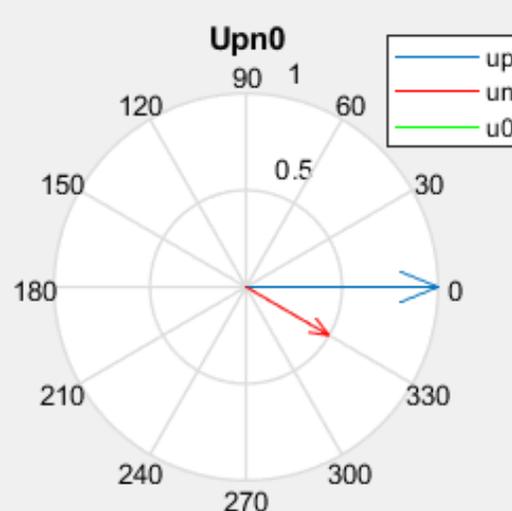
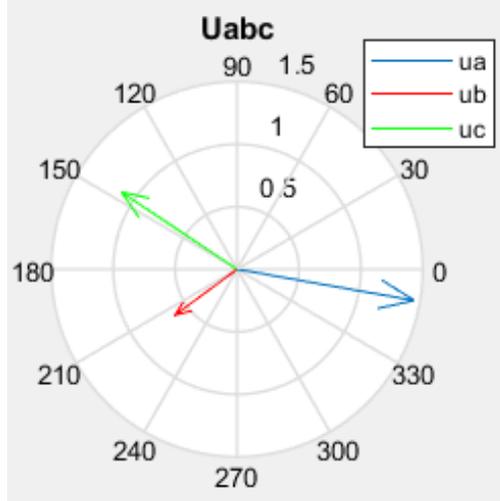
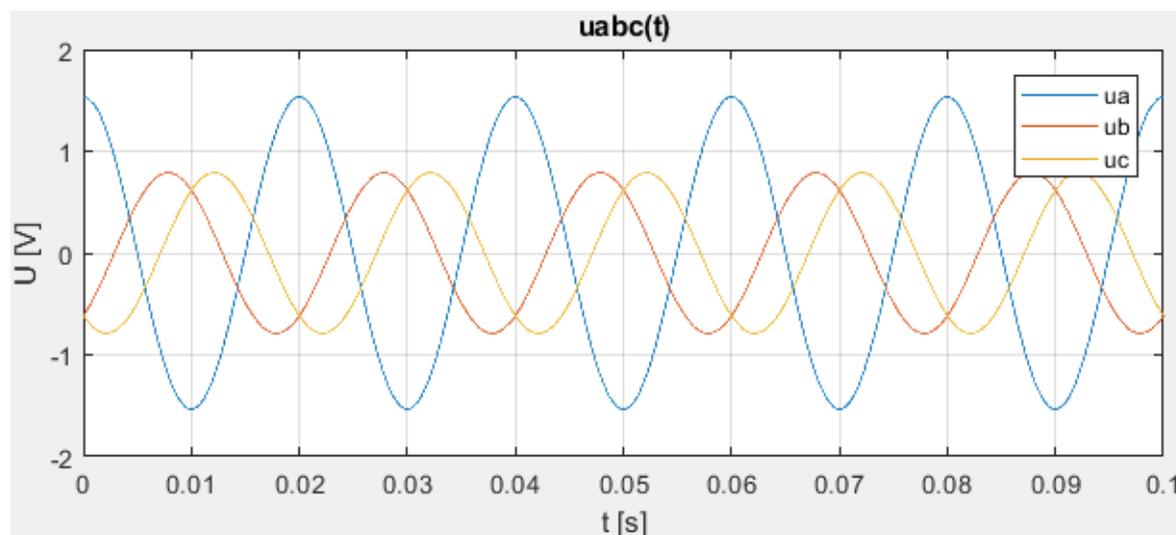


Fig. 2-13 Per-phase circuit and the corresponding phasor diagram.

For symmetrical load analysis of one phase is enough!

Positive/Negative/Zero sequence



Arbitrary 3-phase voltages (or currents) can be composed by:

- **Positive sequence**:
3-phases 120° displaced: a-b-c
Symmetrical
- **Negative sequence**:
3-phases 120° reversely displaced: a-c-b
Backwards rotation
- **Zero sequence**:
3-phases in-phase
 $a+b+c \neq 0$
Current in neutral

Symmetrical components

Positive/Negative/Zero sequence

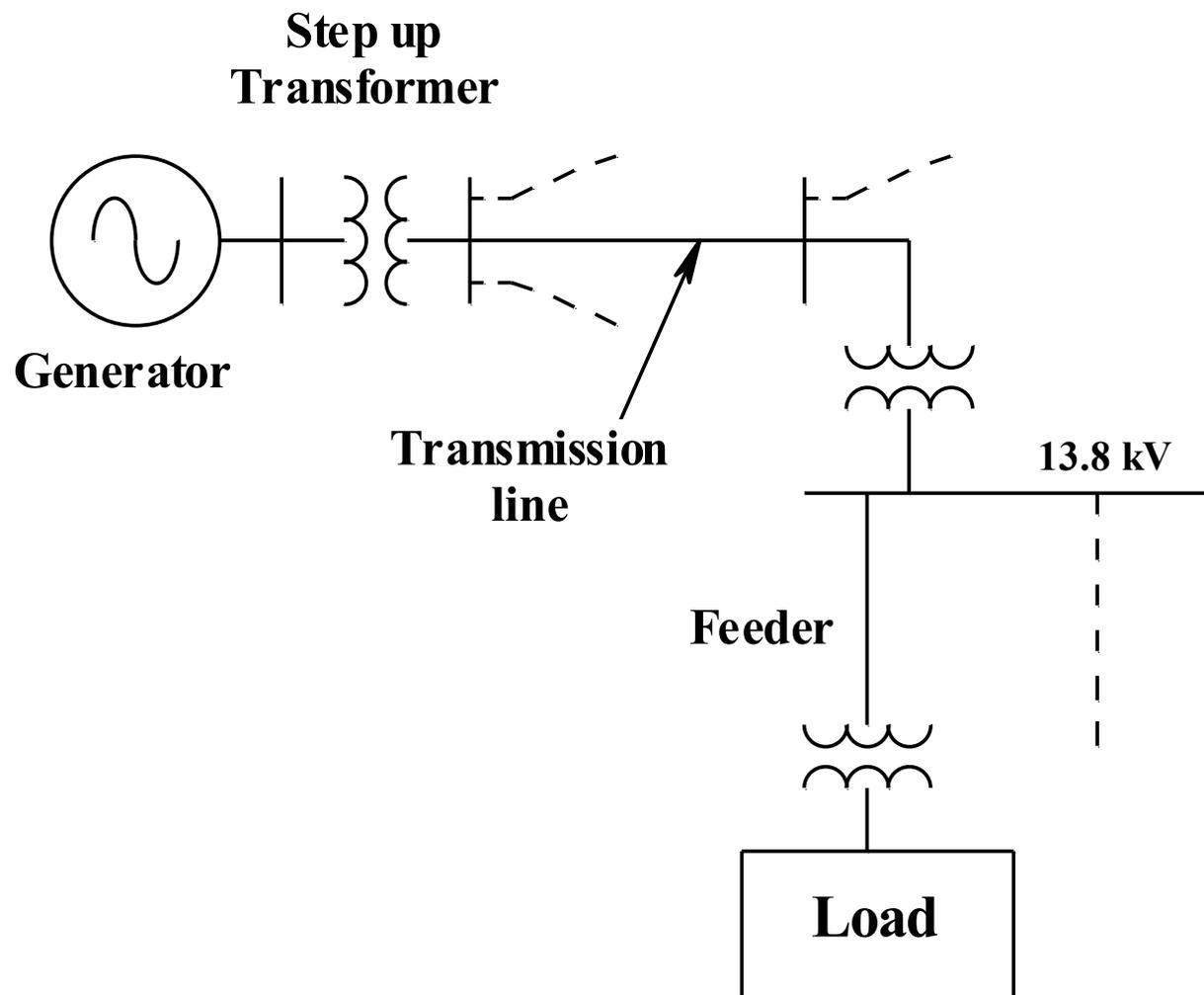
$$\mathbf{V}_{abc} = \begin{bmatrix} \vec{V}_a \\ \vec{V}_b \\ \vec{V}_c \end{bmatrix} = \begin{bmatrix} V_a \angle \theta_a \\ V_b \angle \theta_b \\ V_c \angle \theta_c \end{bmatrix}, \quad \mathbf{V}_{+-0(a)} = \begin{bmatrix} \vec{V}_a^+ \\ \vec{V}_a^- \\ \vec{V}_a^0 \end{bmatrix} = \begin{bmatrix} V_a^+ \angle \theta_a^+ \\ V_a^- \angle \theta_a^- \\ V_a^0 \angle \theta_a^0 \end{bmatrix}$$

$$[T_{+-0}] = \frac{1}{3} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \quad \alpha = e^{j2\pi/3} = 1 \angle 120^\circ$$

$$\mathbf{V}_{+-0(a)} = [T_{+-0}] \mathbf{V}_{abc}$$

- Circuit analysis can now be done separately for positive, negative and zero sequence considering a 1-phase system
- Inverse transformation to abc

One-line (Single-line) Diagram



Phase-Phase (Line-Line) Voltages

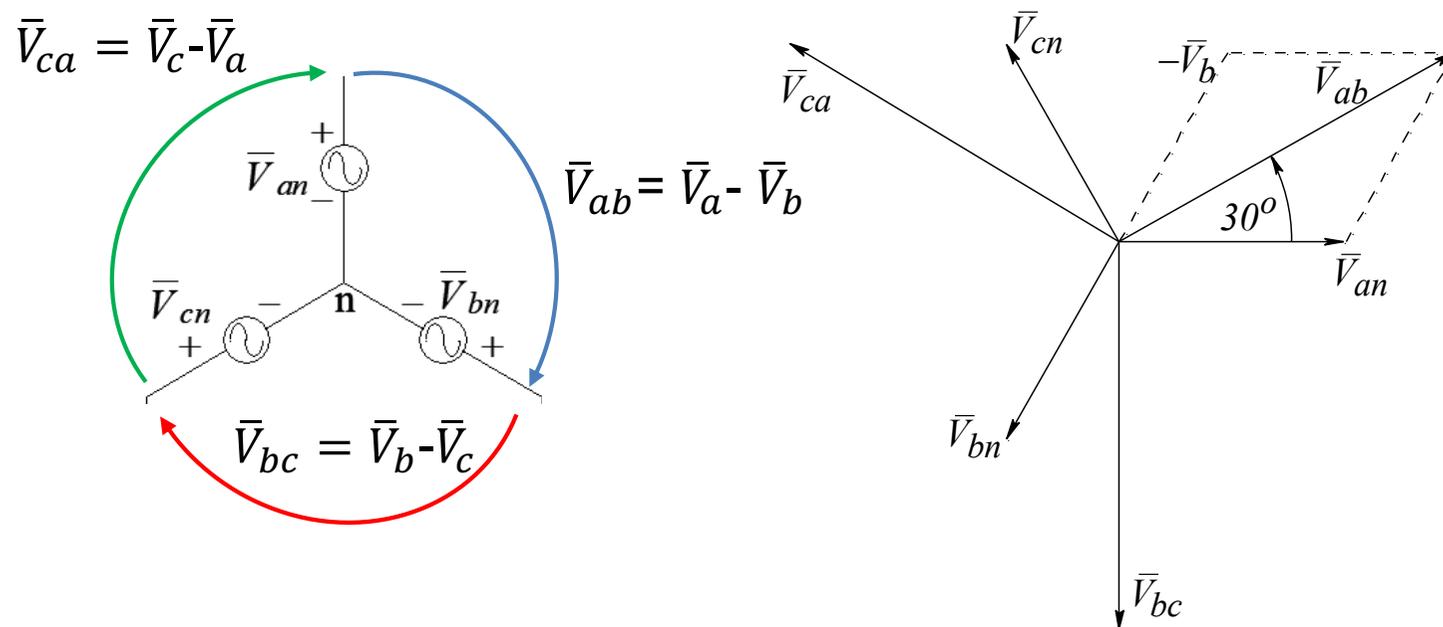


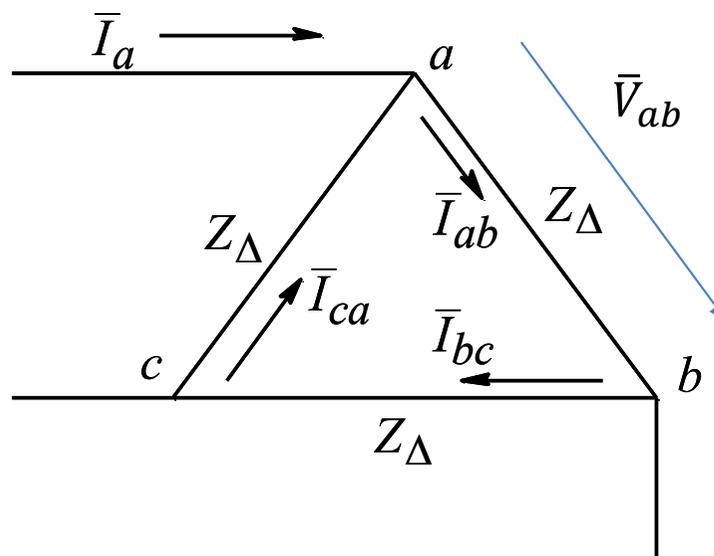
Fig. 2-15 Line-to-line voltages in a three-phase circuit.

Under
symmetrical
conditions:

$$\text{if } \bar{V}_{an} = V \angle 0^\circ$$

$$\bar{V}_{ab} = \sqrt{3} \cdot V \angle 30^\circ$$

Delta Connection



Under
symmetrical
conditions:

$$\begin{aligned} \bar{I}_{ab} &= I \angle 0^\circ \\ \bar{I}_{ca} &= I \angle -240^\circ \\ \bar{I}_a &= \bar{I}_{ab} - \bar{I}_{ca} = \sqrt{3} \cdot I \angle -30^\circ \end{aligned}$$

Power
balance:

$$3\bar{V}_{an} \cdot \bar{I}_a^* = 3\bar{V}_{ab} \cdot \bar{I}_{ab}^*$$

3-phase power

$$P_{3ph} = 3V_{ph}I \cos \phi$$

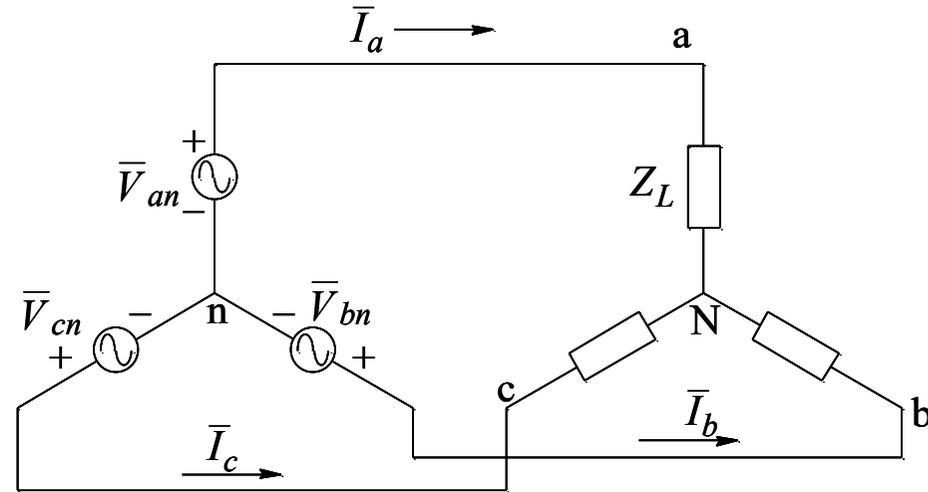
$$Q_{3ph} = 3V_{ph}I \sin \phi$$

$$S_{3ph} = 3V_{ph}I$$

$$\text{Power factor} = \frac{P_{3ph}}{S_{3ph}} = \frac{3V_{ph}I \cos \phi}{3V_{ph}I} = \cos \phi$$

$$V_{ph-ph} = \sqrt{3} \cdot V_{ph}$$

$$P_{3ph} + jQ_{3ph} = \sqrt{3} \cdot \bar{V}_{ph-ph} \cdot \bar{I}^* = \sqrt{3} \cdot \bar{V} \cdot \bar{I}^*$$



(a)

Fig. 2-12 Balanced wye-conne

$\bar{V}_{ph-ph} = \bar{V}$, the normal voltage for describing a 3-phase system

Per Unit Quantities

$$R_{base}, X_{base}, Z_{base} = \frac{V_{base}}{I_{base}} \quad (\text{in } \Omega) \quad (2-48)$$

$$G_{base}, B_{base}, Y_{base} = \frac{I_{base}}{V_{base}} \quad (\text{in } \mathcal{U}) \quad (2-49)$$

$$P_{base}, Q_{base}, (VA)_{base} = V_{base} I_{base} \quad (\text{in Watt, VAR, or VA}) \quad (2-50)$$

In terms of these base quantities, the per-unit quantities can be specified as

$$\text{Per-Unit Value} = \frac{\text{actual value}}{\text{base value}} \quad (2-51)$$

Per Unit Quantities, 3ph

U_{base} =Phase-phase voltage

$$S_{base} = \sqrt{3}U_{base} I_{base}$$

$$Z_{base} = \frac{U_{ph}}{I} = \frac{U_{base}}{\sqrt{3}I_{base}} = \frac{U_{Base}^2}{S_{base}}$$

$$\text{Per-Unit Value} = \frac{\text{actual value}}{\text{base value}}$$

Per unit example

- a) Compute the base current I_{base} and the base impedance Z_{base} given $V_{LL,base} = 208$ and $S_{3ph,base} = 5.4 \text{ kVA}$.
- b) Compute the line current when the voltage is $0.9 \angle 0^\circ \text{ pu}$ and the impedance is $0.5 \angle -60^\circ \text{ pu}$.

Per unit example - solution

$$a) \quad V_{LL,base} = 208 \text{ V}$$
$$V_{ph,base} = \frac{208}{\sqrt{3}} = 120 \text{ V}$$

$$S_{3ph,base} = 5.4 \text{ kVA}$$

$$S_{ph,base} = \frac{5.4}{3} = 1.8 \text{ kVA}$$

$$I_{base} = \frac{S_{ph,base}}{V_{ph,base}} = \frac{S_{3ph,base}}{\sqrt{3}V_{LL,base}} = 15 \text{ A}$$

$$Z_{base} = \frac{V_{ph,base}}{I_{base}} = \frac{V_{LL,base}^2}{S_{3ph,base}} = 8 \text{ ohm}$$

$$b) \quad \bar{v} = 0.9 \angle 0^\circ \text{ pu}$$

$$\bar{z} = 0.5 \angle -60^\circ \text{ pu}$$

$$\bar{i} = \frac{\bar{v}}{\bar{z}} [\text{pu}] = 0.99 + j1.56$$

$$|\bar{i}| = 1.8 \text{ pu}$$

$$\bar{I} = \bar{i} \cdot I_{base} [\text{A}] = 1.35 + j23.4 \text{ A} = 27 \angle 60^\circ \text{ A}$$

AC Transmission lines

Transmission Network in Nordic

Transmission lines:
220-400 kV

Regional lines:
40-130 kV

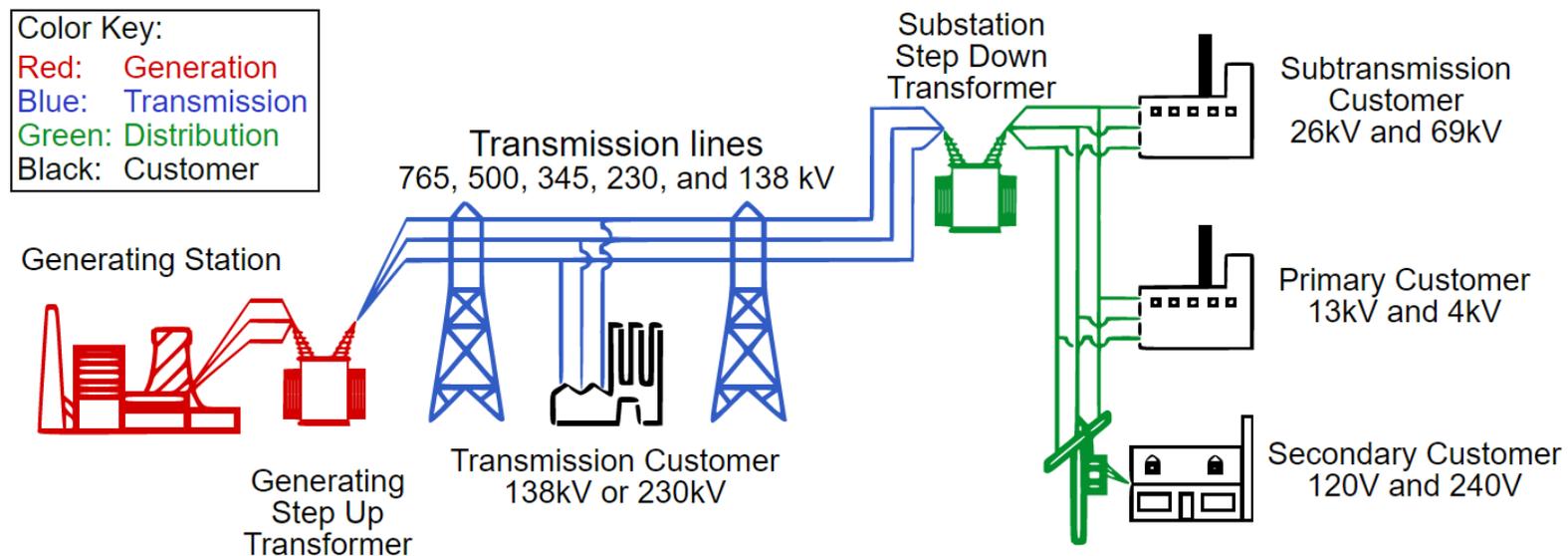
Distribution lines:
10-20 kV

Images:
Svenska kraftnät
Vattenfall



Overhead AC Transmission Lines

- Distribution Lines
 - 10 kV, 36 kV
- Transmission Line Voltages (Web link: [SVK](#))
 - 115 kV, 230 kV, 345 kV, 500 kV and 765 kV
- Three-Phase



Transmission Tower, Conductor and Bundling

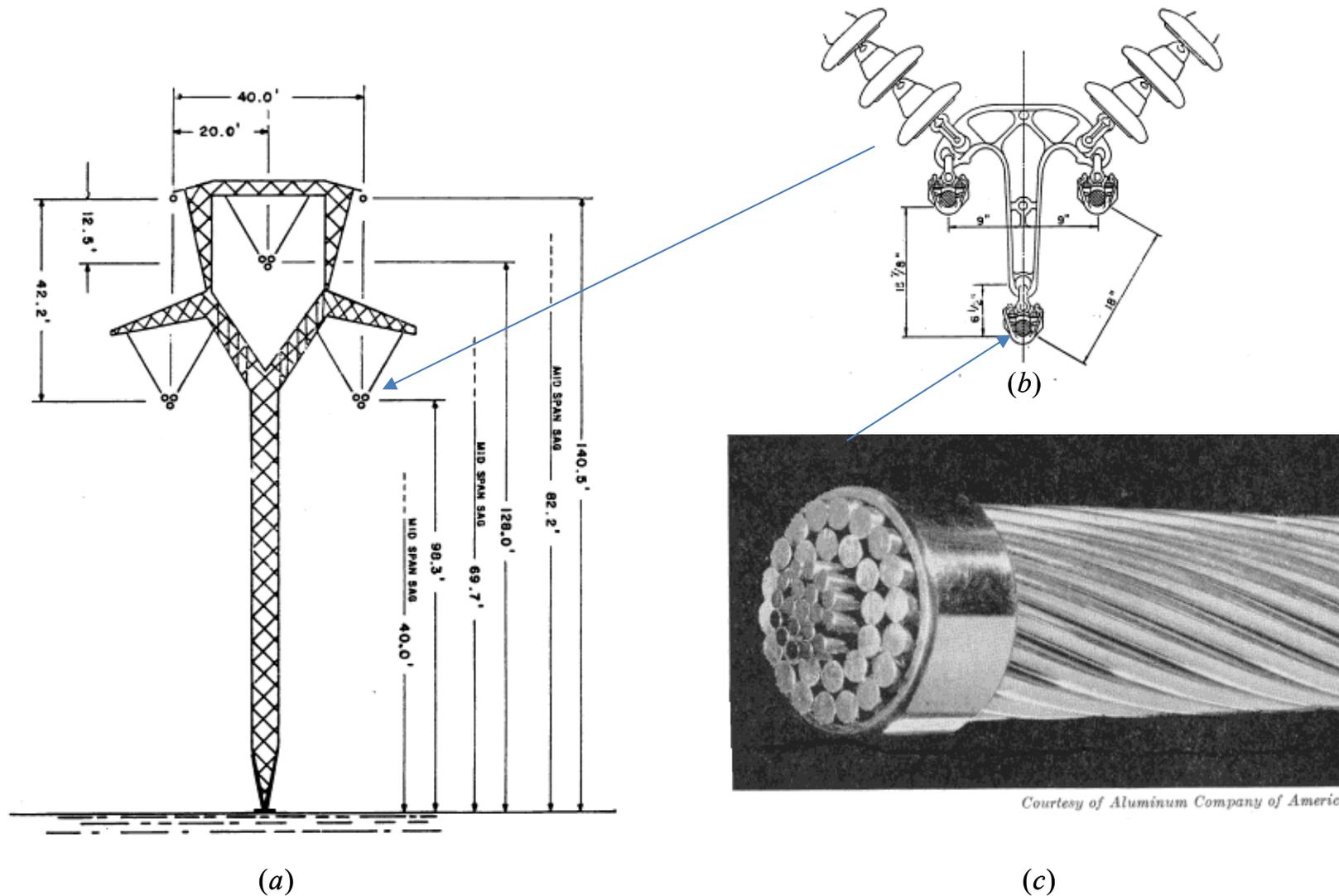
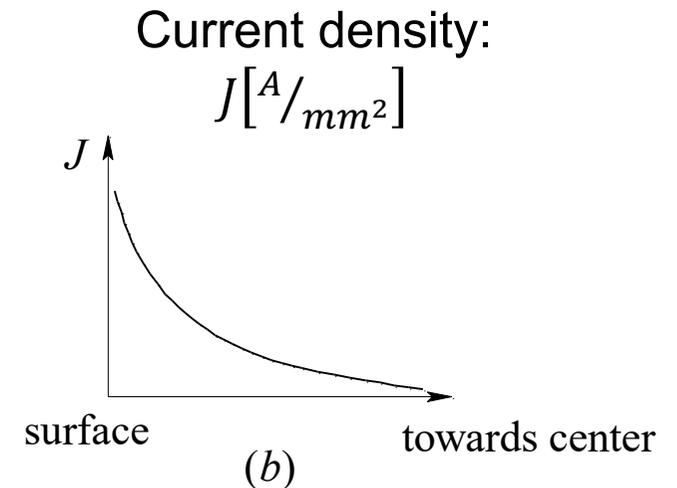
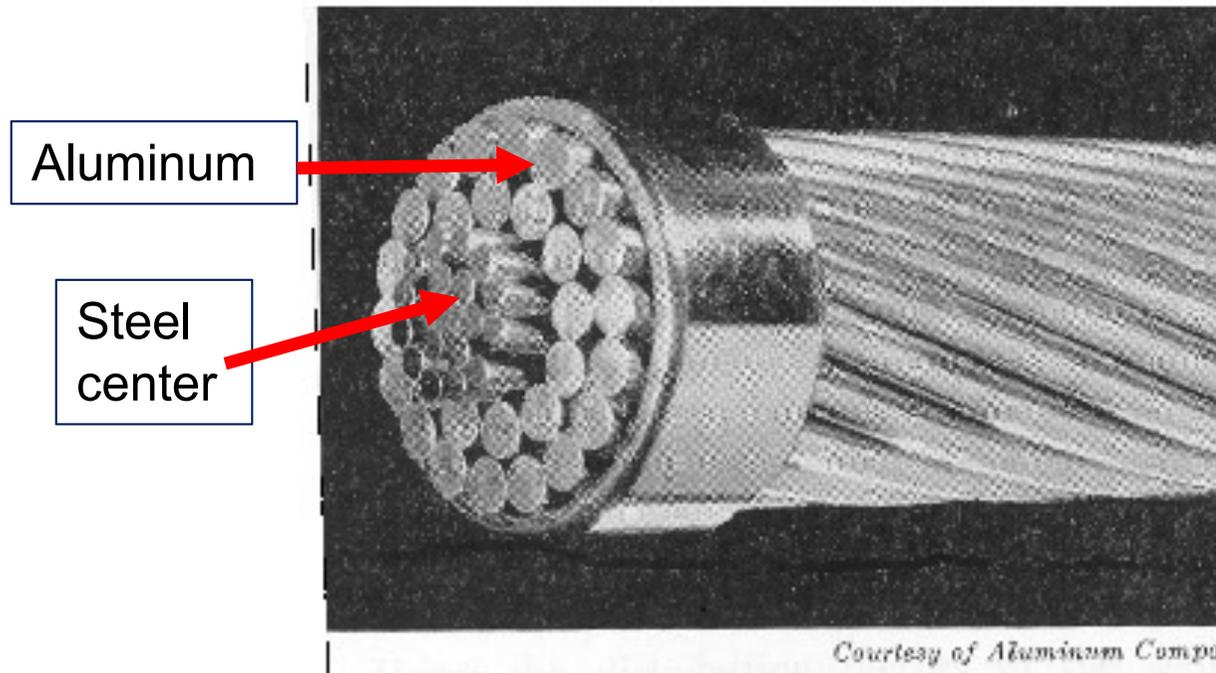


Fig. 4-1 500-kV transmission line (Source: University of Minnesota EMTP course).

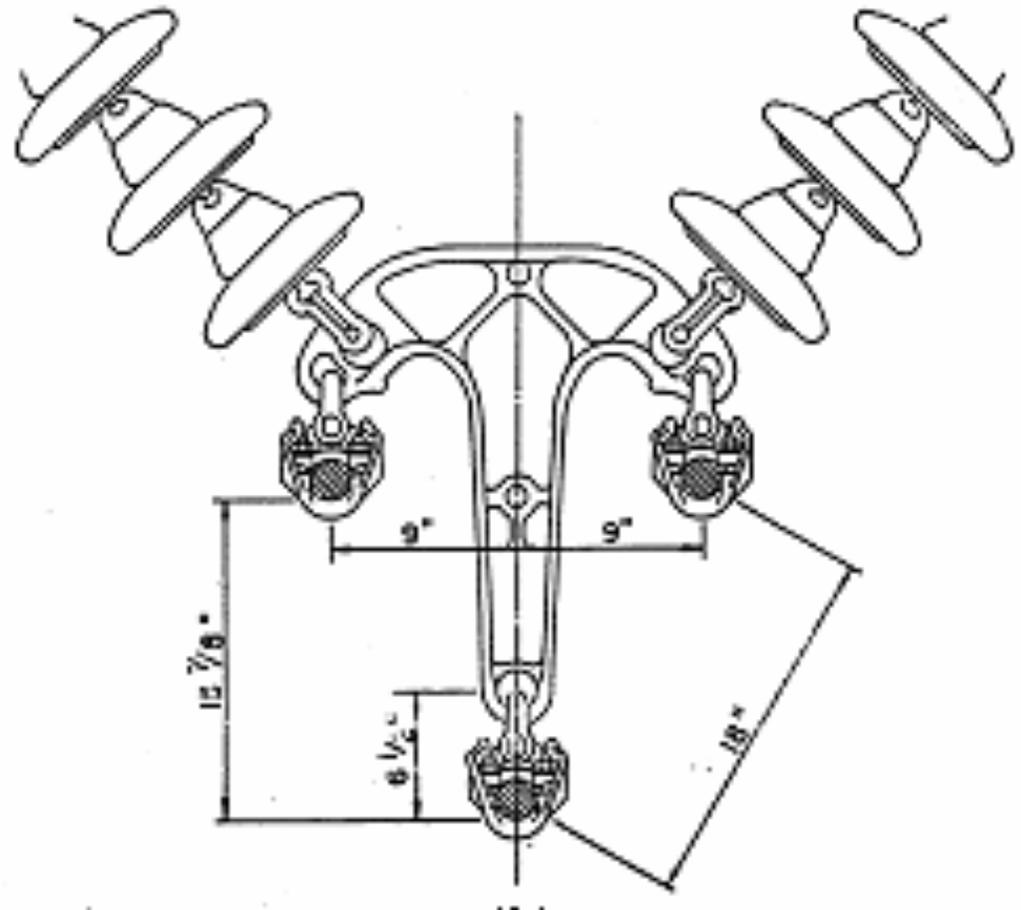
Conductors:



- ACSR, **A**luminum **C**onductor, **S**teel **R**einforced
- Skin effect = current confined to the surface
 - Aluminum in the outer part for low resistance
 - Steel in the center for mechanical strength

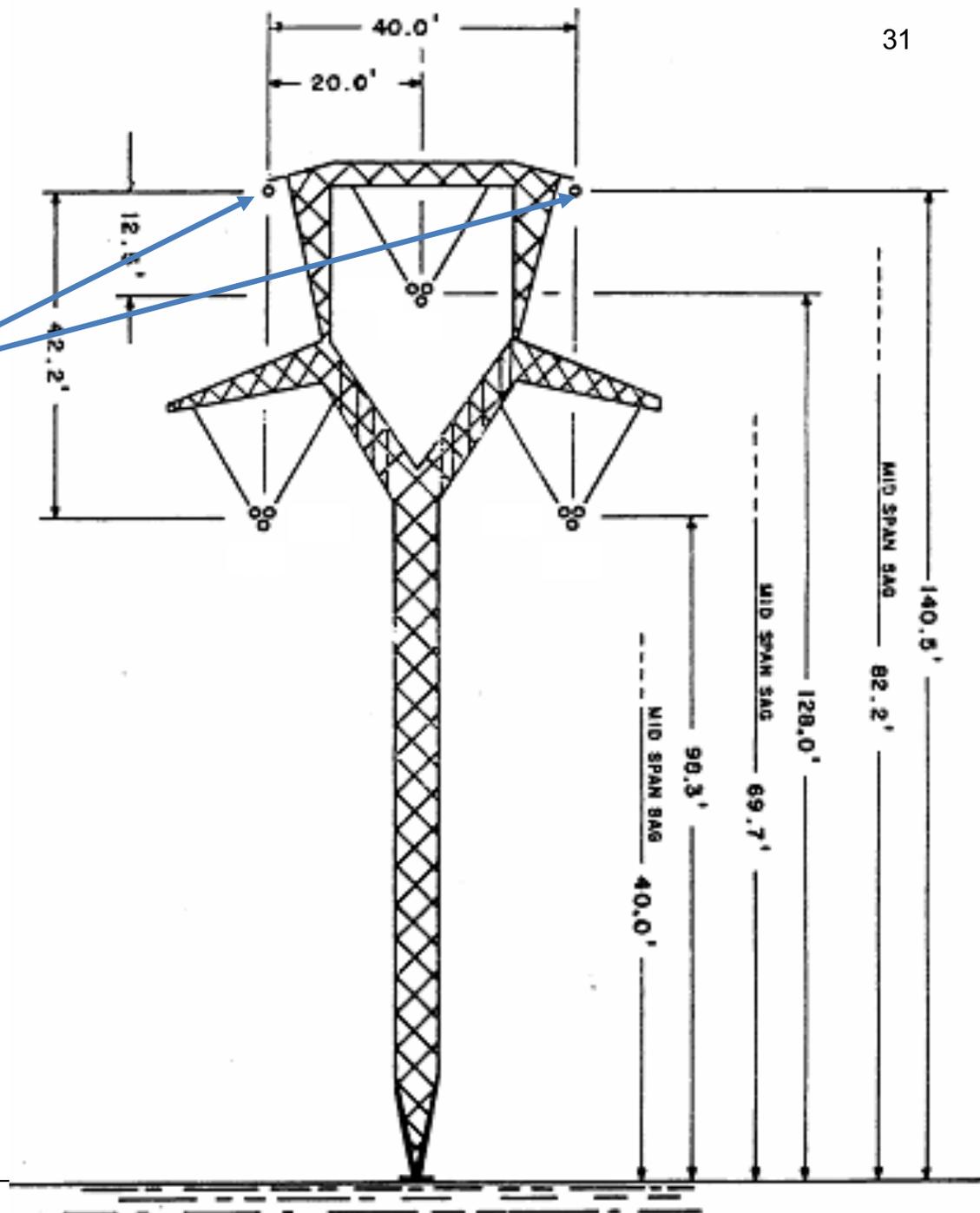
Bundling:

- To reduce electric field at the conductor surface
- Less than 16 kV/cm
- 345 kV Lines
 - 2 conductor-bundle at 18 inches
- 500 kV Lines
 - 3 conductor bundle at 18 inches

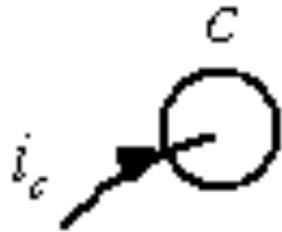


Shield Wires

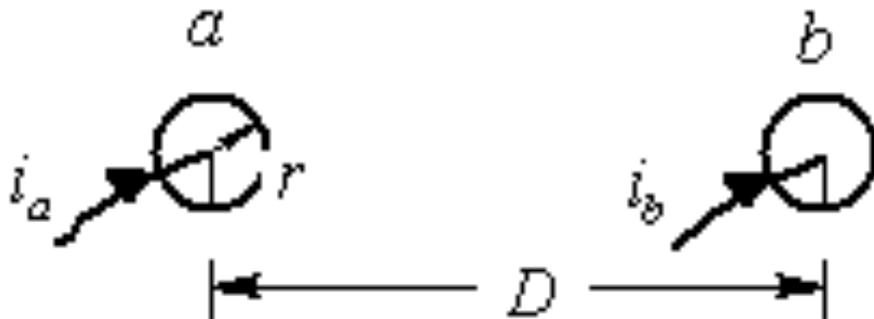
- Grounded wires at the top of the tower
- Shielding of Lightning strikes



Transmission Line Inductance



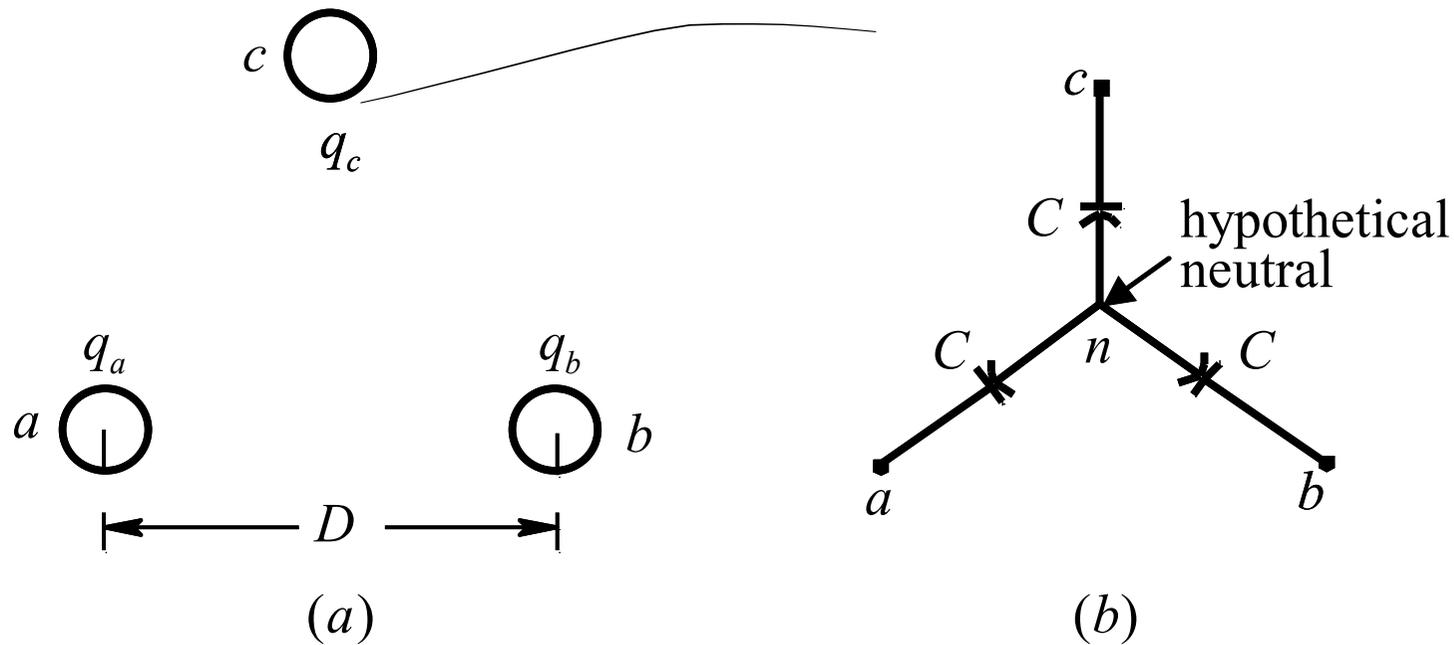
$$i_a + i_b + i_c = 0$$



$$L = \left(\frac{\mu_0}{2\pi} \right) \ln \frac{D}{r}$$

GMD $D = \sqrt[3]{D_1 D_2 D_3}$

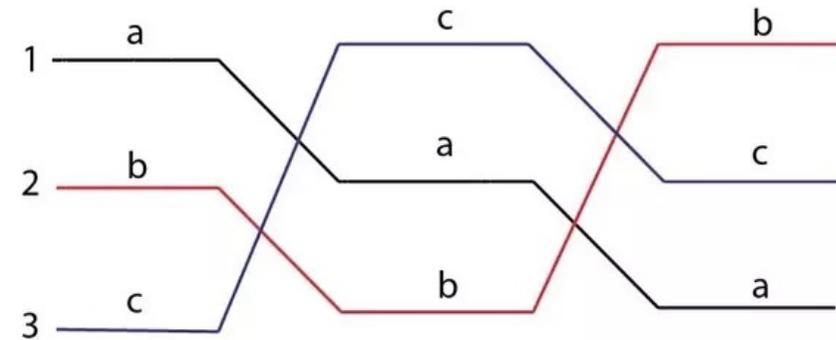
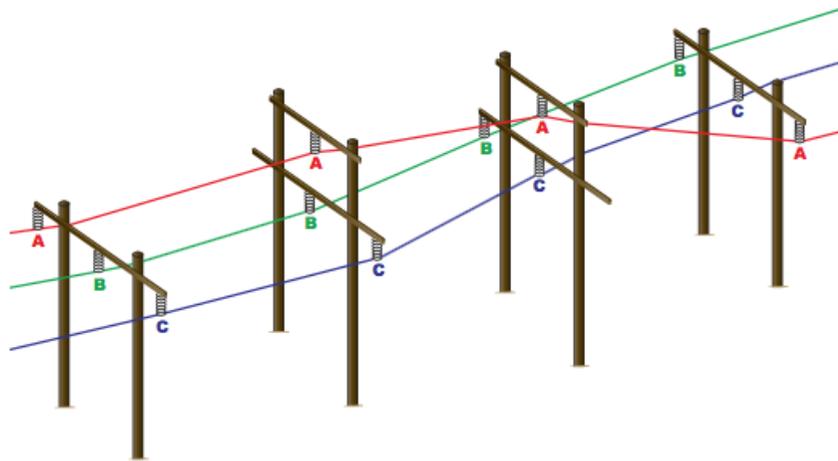
Transmission Line Capacitance



$$C = \frac{2\pi\epsilon_0}{\ln \frac{D}{r}}$$

$$D = \sqrt[3]{D_1 D_2 D_3}$$

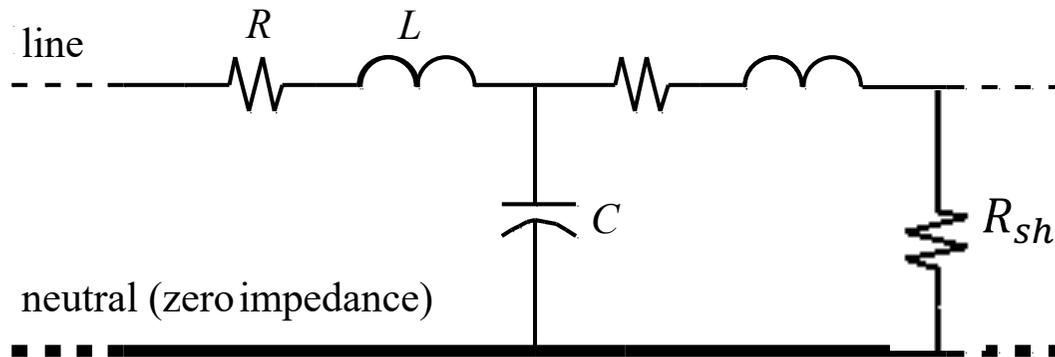
Transposition



Transposition of the Conductors

- Conductors in different positions gets different L and C
- Transposition neutralizes the difference over the length of the line

Transmission Line Parameters:



- Distributed Parameters per km
- Analyzed on a per-phase basis

$$\bar{U} = \bar{Z} \cdot \bar{I} \quad \text{Impedance: } \bar{Z} = R + jX \text{ [ohm]}$$

$$\text{Reactance (ind): } X_L = \omega L \text{ [ohm]} \quad \text{Reactance (cap): } X_C = \frac{-1}{\omega C} \text{ [ohm]}$$

$$\bar{I} = \bar{Y} \cdot \bar{U} \quad \text{Admittance: } \bar{Y} = \frac{1}{\bar{Z}} = G + jB \text{ [mho or S]}$$

$$\text{Conductance: } G = \frac{1}{R_{sh}} \text{ [mho]} \quad \text{Susceptance: } B = \frac{-1}{X_C} = \omega C \text{ [mho]}$$

Typical Parameters for various Voltage Transmission Lines

Table 4-1

Transmission Line Parameters with Bundled Conductors (except at 230 kV)
at 60 Hz [2, 6]

Nominal Voltage	$R (\Omega / km)$	$\omega L (\Omega / km)$	$\omega C (\mu mho / km)$
230 kV	0.055	0.489	3.373
345 kV	0.037	0.376	4.518
500 kV	0.029	0.326	5.220
765 kV	0.013	0.339	4.988

Reactance (ind): $X_L = \omega L$

Susceptance: $B = \omega C$

Voltage Profile With Surge Impedance Loading (SIL)

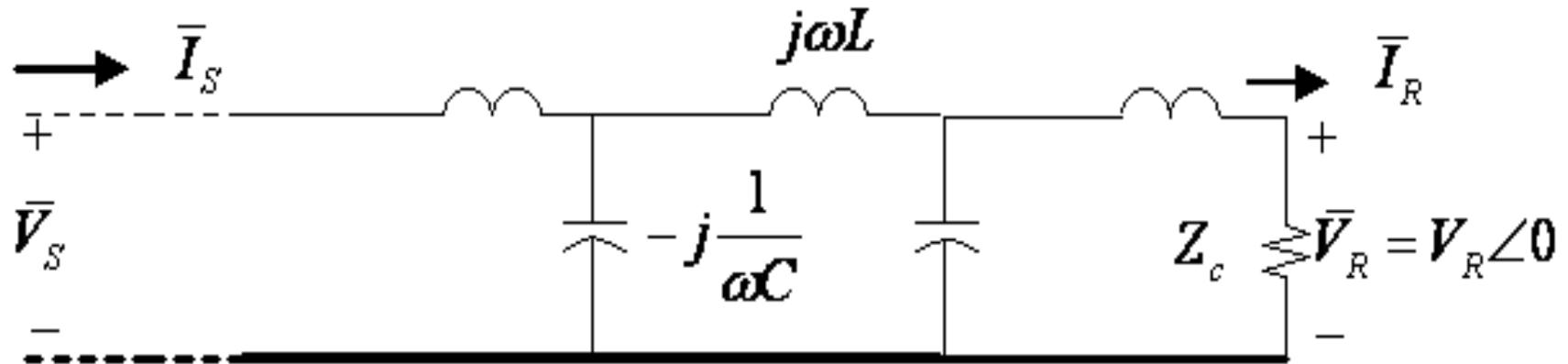


Fig. 4-10 Per-phase transmission line terminated with a resistance equal to Z_c .

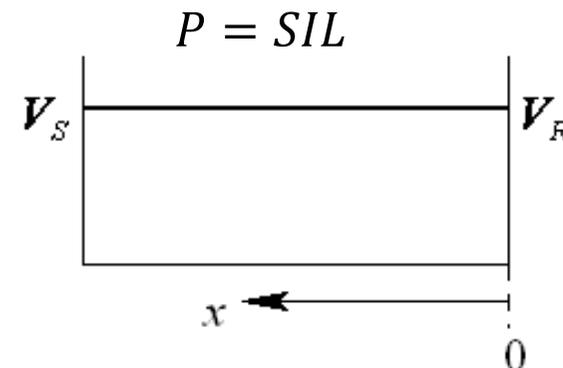
Reactive power balance:

$$\omega L I_x^2 = V_x^2 \omega C$$

Surge impedance:

$$Z_c = \sqrt{\frac{L}{C}}$$

$$SIL = \frac{V_{LL}^2}{Z_c}$$



Typical Surge Impedances and SIL for various Voltage Transmission Lines

Table 4-2

Surge Impedance and Three-Phase Surge Impedance Loading [2, 6]

Nominal Voltage	$Z_c (\Omega)$	$SIL (MW)$
230 kV	375	140 MW
345 kV	280	425 MW
500 kV	250	1000 MW
765 kV	255	2300 MW

Loadability of Transmission Lines

Table 4-3
Loadability of Transmission Lines [6]

Line Length (km)	Limiting Factor	Multiple of SIL
0 - 80	Thermal	> 3
80 - 240	5% Voltage Drop	1.5 - 3
240 - 480	Stability	1.0 – 1.5

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